Math 54 Handout 4

June 21, 2018

Question 1.

$$T\begin{pmatrix} 5 \\ -3 \end{pmatrix} = T\left(5\begin{pmatrix} 1 \\ 0 \end{pmatrix} + (-3)\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = 5T\begin{pmatrix} 1 \\ 0 \end{pmatrix} + (-3)T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 5y_1 - 3y_2$$

Similarly

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1y_1 + x_2y_2$$

Question 2.

No. We check the property $T(cv) = cT(v)$.

$$T\left(c\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) = T\begin{pmatrix} cx_1 \\ cx_2 \end{pmatrix} = \begin{pmatrix} 4cx_1 - 2cx_2 \\ 3|cx_2| \end{pmatrix} \neq \begin{pmatrix} 4cx_1 - 2cx_2 \\ 3c|x_2| \end{pmatrix} = cT\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Question 3. $T : \mathbb{R} \to \mathbb{R}$ takes

$$T(a) = T(a * 1) = T(1) * a$$

so the image of a is just a scalar (here it is $T(1)$) multiplied by a. Thus all linear transformations from a one dimensional vector space to itself is multiplication by some scalars.

Question 4.

The linear transformation takes $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ to $\begin{pmatrix} cos(\theta) \\ sin(\theta) \end{pmatrix}$ and takes $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ to $\begin{pmatrix} -sin(\theta) \\ cos(\theta) \end{pmatrix}$, so the matrix looks like

$$\begin{pmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{pmatrix}$$

Question 5.

True or False:

1. True.

$$\begin{aligned}(T_1 \circ T_2)(v_1 - cv_2) &= T_1(T_2(v_1 - cv_2)) = T_1(T_2(v_1) - cT_2(v_2)) \\ &= T_1(T_2(v_1)) - cT_1(T_2(v_2)) = (T_1 \circ T_2)(v_1) - c(T_1 \circ T_2)(v_2)\end{aligned}$$

2. False. It can have n pivots, making it one-to-one.

3. True. It cannot have m pivots as $m > n$.

Question 6.

Putting the vectors into a matrix A, we obtain a $m \times n$ matrix. Since the columns of A span $\mathbb{R}^m$, there must be m pivots, and hence $n \geq m$. On the other hand, if the columns of A are independent, then there must be n pivots, and $n \leq m$.