Math 54 Handout 4

June 19, 2018

Question 1.

Let $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, y_1 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, and $y_2 = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps e_1 to y_1 and e_2 to y_2. Find the image of $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

Question 2.

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by sending $(x_1, x_2) \mapsto (4x_1 - 2x_2, 3|x_2|)$. Is T a linear transformation?

Question 3.

Show that every linear map from a one-dimensional vector space to itself is multiplication by some scalar.

Question 4.

Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotates points about the origin by an angle θ, what is the standard matrix for T?

Question 5.

True or False:

1. The composition of two linear transformations is still a linear transformation.
2. Suppose $m > n$. A $m \times n$ matrix cannot be one to one.
3. Suppose $m > n$. A $m \times n$ matrix cannot be onto.

Question 6.

Suppose $\{v_1, ..., v_n\}$ span $\mathbb{R}^m$, what can we say about n and m? Suppose $\{v_1, ..., v_n\}$ are linearly independent in $\mathbb{R}^m$, what can we say about n and m?