Math 54 Section 701 Handout 2 Solution

June 19, 2018

Question 1.

$$x_1 \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix} + x_3 \begin{pmatrix} 5 \\ -1 \\ -8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Question 2.

$$\begin{pmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 5 & 4 & 3 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

which is inconsistent, so b is not a linear combination of a_1, a_2, and a_3.

Question 3.

$$\begin{pmatrix} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 3 & h+8 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 3 & h+8 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & h+17 \end{pmatrix}$$

In order for b to be in the plane spanned by a_1 and a_2, the above matrix must be consistent, and hence $h + 17 = 0$, so $h = -17$.

Question 4.

$$\begin{pmatrix} 4 & 5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \\ 0 \end{pmatrix}$$

Question 5.

$$\begin{pmatrix} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 5 & 8 & 7 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & -7 & 7 & -8 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & -29 \end{pmatrix}$$

which is not consistent, so u is not in the subset of $\mathbb{R}^3$ spanned by the columns of A.

Question 6.

False, as a 5×3 matrix can have at most three pivots, so there will be at least 2 rows without pivots.

Question 7.

(Ignore this question for 6/19, I will cover this material on 6/20)

True. Since $Ax = b$ has a unique solution, we see that there must be a pivot in every column (no free variables). But A is 3×3, so we have a pivot for each row.