Math 54 Handout 1 Solution

June 19, 2018

Question 1.

We turn this system into the following matrix $\begin{pmatrix} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{pmatrix}$ and row reduce.

$$\begin{pmatrix} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 3 & 7 & 7 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & -2 & -8 & 12 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

which is inconsistent because $0 \neq 2$.

Question 2.

Row reduction gives

$$\begin{pmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{pmatrix} \to \begin{pmatrix} 1 & h & 4 \\ 0 & 6-3h & -4 \end{pmatrix}$$

so we see that this is consistent if and only if $6 - 3h \neq 0$, or equivalently $h \neq 2$.

Question 3.

Doing row reduction on the c position, we reduce the above matrix to $\begin{pmatrix} 1 & 3 & u \\ 0 & d-3c & v-3u \end{pmatrix}$. Thus this has solution for all u, v if and only if this matrix is consistent for all u, v. This forces $d - 3c \neq 0$, which is the same as saying that $d \neq 3c$.

Question 4.

$$\begin{pmatrix} 1 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & -1 & 3 \\ 0 & 0 & 1 & -7 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 0 & -4 \\ 0 & 0 & 1 & -7 \end{pmatrix}$$

Question 5.

$$\begin{pmatrix} 0 & 1 & -6 & 5 \\ 1 & -2 & 7 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 7 & -6 \\ 0 & 1 & -6 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -5 & 4 \\ 0 & 1 & -6 & 5 \end{pmatrix}$$

so writing it back as system of equations, we have $x_1 - 5x_3 = 4$ and $x_2 - 6x_3 = 5$. Thus expressing x_1, x_2 in terms of free variables x_3, we get $x_1 = 4 + 5x_3$ and $x_2 = 5 + 6x_3$.

Question 6.

It is always consistent. There can be at most 3 pivots for a 3×5 matrix, and the problem states that all three of those appear in the coefficient matrix. So when augmented with any right hand side, the last column (the right hand side) cannot have any pivots. Thus it is consistent.