Math 54 Handout 18

August 3, 2018

Question 1.

The characteristic polynomial of the matrix A is $(r-1)^2$, so the eigenvalue 1 has multiplicity 2. For eigenvalue 1, $A - rI = \left(\begin{array}{cc|c} -1 & 1 & 0 \\ -1 & 1 & 0 \end{array}\right)$ which has eigenvector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. To find a generalized eigenvector, we solve $\left(\begin{array}{cc|c} -1 & 1 & 1 \\ -1 & 1 & 1 \end{array}\right)$ which has a generalized eigenvector $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$. The corresponding solutions are $e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $e^t \left(\begin{pmatrix} -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right)$.

Question 2.

The characteristic polynomial for A is $r(r-1)^2$ where the roots are 0 and 1. For $r = 0$, the corresponding eigenvector is $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, and for $r = 1$, the corresponding eigenvector is given by solving $\left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$, which gives eigenvector $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. A generalized eigenvector is given by solving $\left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$ which gives $\begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix}$. The solutions corresponding to these are $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $e^t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, and $e^t \left(\begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right)$