Math 54 Handout 16

July 27, 2018

Question 1.

1. The roots of the polynomial $r^2 - r - 11 = 0$ are $r = \frac{1 \pm \sqrt{45}}{2}$, so the general solution is the span of $e^{\frac{1+\sqrt{45}}{2}t}$ and $e^{\frac{1-\sqrt{45}}{2}}t$.

2. The roots of the polynomial $r^2 + 2r + 5 = 0$ are $\frac{-2 \pm \sqrt{-16}}{2} = -1 \pm 2i$, and so the general solution is the span of $e^{-t}cos(2t)$ and $e^{-t}sin(2t)$.

3. $y'' - 2y' + y = 8e^t$. The root of the polynomial $r^2 - 2r + 1 = 0$ is $r = 1$, so the homogeneous solution is the span of e^t and te^t. The particular solution we get by using method of undetermined coefficients. Here $m = 0, r = 1$, and $s = 2$, so the form we guess is $y_p = At^2e^t$. Now $y_p' = At^2e^t + 2Ate^t$, and $y_p'' = At^2e^t + 4Ate^t + 2Ae^t$. Plugging in the original differential equation, we get

$$(At^2e^t + 4Ate^t + 2Ae^t) - 2(At^2e^t + 2Ate^t) + At^2e^t = 8e^t$$

   Simplifying we get $2Ae^t = 8e^t$, so $A = 4$. So the particular solution is $4t^2e^t$, and hence the general solution is $4t^2e^t + c_1e^t + c_2te^t$.

4. $y'' - 2y' + y = t^{-1}e^t$. We use variation of parameters here. The homogeneous solution is the span of $y_1 = e^t$ and $y_2 = te^t$ (same as the previous part). The Wronskian is e^{2t}. Now

$$v_1' = \frac{-t^{-1}e^t \cdot te^t}{1 \cdot e^{2t}} = -1, \quad v_2' = \frac{t^{-1}e^t \cdot e^t}{1 \cdot e^{2t}} = t^{-1}$$

   so we see that

$$v_1 = -t, \quad v_2 = ln|t|$$

   so we conclude that the particular solution is

$$-te^t + ln|t|te^t$$

   and hence the general solution is $-te^t + ln|t|te^t + c_1e^t + c_2te^t$.