

Math 54 Handout 13

July 16, 2018

Question 1.

Let $y = \begin{pmatrix} 4 \\ 8 \\ 1 \end{pmatrix}$, $u_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \\ 3 \end{pmatrix}$, $u_2 = \begin{pmatrix} -2 \\ 1 \\ 3 \\ 3 \end{pmatrix}$ and let $W = \text{Span}\{u_1, u_2\}$.

1. Let $U = (u_1 \ u_2)$. Compute $U^T U$ and $U U^T$.
2. Compute $\text{Proj}_W y$ and $U U^T y$.

Question 2.

Let W be a subspace of $\mathbb{R}^n$ with an orthogonal basis $\{w_1, \dots, w_p\}$ and let $\{v_1, \dots, v_q\}$ be an orthogonal basis for $W^\perp$.

1. Explain why $\{w_1, \dots, w_p, v_1, \dots, v_q\}$ is an orthogonal set.
2. Explain why the above set span $\mathbb{R}^n$.
3. Show that $\dim(W) + \dim(W^\perp) = n$.

Question 3.

Find an orthogonal basis for the column space of the matrix

$$\begin{pmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{pmatrix}$$

Question 4.

Find a least square solution to $Ax = b$, where $A = \begin{pmatrix} -1 & 2 \\ 2 & -3 \\ -1 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$