

Math 54 Handout 12

July 9, 2018

Question 1.

Show that $W \cap W^\perp = \{0\}$.

Question 2.

Show that the set $\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \right\}$ is orthogonal. Express $\begin{pmatrix} 8 \\ -4 \\ -3 \end{pmatrix}$ as a linear combination of elements in the orthogonal set.

Question 3.

Is the matrix $\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ orthogonal? How about the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix}$?

Question 4.

Show that the determinant of an orthogonal matrix is always ± 1 .

Question 5.

Suppose A is an orthogonal matrix, then x and Ax have the same length.

Question 6.

Let W be the subspace spanned by the u 's and write y as a sum of a vector in W and a vector orthogonal to W , where

$$y = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}, u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$$