Math 54 Handout 10

July 11, 2018

Question 1.

$$det\begin{pmatrix} 3-\lambda & -2 & 5 \\ 1 & 0-\lambda & 7 \\ 0 & 0 & 2-\lambda \end{pmatrix} = (2-\lambda)((3-\lambda)(-\lambda)+2) = (2-\lambda)(\lambda-1)(\lambda-2)$$

So the eigenvalues are 2 with multiplicity 2 and 1 with multiplicity 1.

Question 2.

$$det\begin{pmatrix} 3-\lambda & 0 & -2 \\ -7 & 0-\lambda & 4 \\ 4 & 0 & 3-\lambda \end{pmatrix} = -\lambda(\lambda^2-6\lambda+17)$$

It is not diagonalizable over the real numbers because there is only one real eigenvalue with one corresponding real eigenvector.

Question 3.

1. $\begin{pmatrix} T(e_1) & T(e_2) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

2. $\begin{pmatrix} [T(b_1)]_B & [T(b_2)]_B \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix}_B & \begin{pmatrix} 3 \\ 1 \end{pmatrix}_B \end{pmatrix}$. To find this matrix, augment it to the matrix $\begin{pmatrix} b_1 & b_2 \end{pmatrix}$ and solve

$$\left(\begin{array}{cc|cc} 1 & 1 & -1 & 3 \\ -1 & 3 & 1 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 1 & -1 & 3 \\ 0 & 4 & 0 & 4 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 1 & -1 & 3 \\ 0 & 1 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & 1 \end{array}\right)$$

and so $[T]_B = \begin{pmatrix} [T(b_1)]_B & [T(b_2)]_B \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix}_B & \begin{pmatrix} 3 \\ 1 \end{pmatrix}_B \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$

3. Similar matrix have the same determinants. However, the determinant of the matrix in part (a) is -1, and the identity matrix here has determinant -1, they cannot be similar, and hence there can be no basis $\mathfrak{A}$ such that $[T]_{\mathfrak{A}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Question 4.

$$[T]_{\mathfrak{B}} = \begin{pmatrix} [T(b_1)]_{\mathfrak{B}} & [T(b_2)]]_{\mathfrak{B}} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix}_{\mathfrak{B}} & \begin{pmatrix} 11 \\ -3 \end{pmatrix}_{\mathfrak{B}} \end{pmatrix}$$

To calculate the last matrix above, we compute the following:

$$\left(\begin{array}{cc|cc} 2 & 1 & 2 & 11 \\ -1 & 2 & -1 & -3 \end{array}\right) \to \left(\begin{array}{cc|cc} 2 & 1 & 2 & 11 \\ 0 & 5 & 0 & 5 \end{array}\right) \to \left(\begin{array}{cc|cc} 2 & 1 & 2 & 11 \\ 0 & 1 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 2 & 0 & 2 & 10 \\ 0 & 1 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & 1 \end{array}\right)$$

and so $[T]_{\mathfrak{B}} = \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$

Question 5.

True of False:

1. True. $A(Bv) = ABv = BAv = B\lambda v = \lambda(Bv)$.
2. False. Counterexample: $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$
3. True. Complex roots of a (characteristic) polynomial come in conjugate pairs.