Math 54 Handout 10

July 9, 2018

Question 1.

Let

$$A = \begin{pmatrix} 3 & -2 & 5 \\ 1 & 0 & 7 \\ 0 & 0 & 2 \end{pmatrix}$$

Find all real eigenvalues of A and their multiplicities.

Question 2.

Let

$$A = \begin{pmatrix} 3 & 0 & -2 \\ -7 & 0 & 4 \\ 4 & 0 & 3 \end{pmatrix}$$

Is this matrix diagonalizable? Why or why not?

Question 3.

Suppose that in the standard basis of $\mathbb{R}^2$, the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is given by

$$T \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a \end{pmatrix}$$

1. Find the matrix representation of T in the standard basis.
2. If $\mathfrak{B} = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$, find $[T]_{\mathfrak{B}}$.
3. Show that there can be no basis $\mathfrak{A}$ such that $[T]_{\mathfrak{A}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Question 4.

Suppose that in the standard basis of $\mathbb{R}^2$, the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is given by the matrix $\begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix}$. If $\mathfrak{B} = \left\{ \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$, find $[T]_{\mathfrak{B}}$.

Question 5.

True of False:

1. If A and B commute, and if v is an eigenvector of A with eigenvalue λ, then Bv is also an eigenvector of A with eigenvalue λ.
2. If A^2 is diagonalizable, then so is A.
3. A 3×3 matrix has always one real eigenvalue.