Homework # 5

August 1st
Due: August 6th, 2013

Exercise 1.

Find the flux of the vector field $\vec{F} = \langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \rangle$ outwards through any circle centered at $(1, 0)$ of radius $r \neq 1$.

Exercise 2.

Evaluate the surface integral $\int\int_\Sigma \vec{F}\cdot\hat{\mathbf{n}}\, dS$ where Σ is the part of the cone $z = \sqrt{x^2+y^2}$ beneath the plane $z = 1$ with downward orientation and $\vec{F} = \langle x, y, z^4 \rangle$.

Exercise 3.

Verify Stokes' theorem is true for $\vec{F} = \langle y, z, x \rangle$ with surface Σ the hemisphere $x^2 + y^2 + z^2 = 1$, $y \geq 0$, oriented in the positive y direction.

Exercise 4.

(a) Let $f(x, y, z) = 1/\rho = (x^2 + y^2 + z^2)^{-1/2}$. Calculate $\vec{F} = \nabla f$, and describe geometrically the vector field $\vec{F}$.
(b) Evaluate the flux of $\vec{F}$ over the sphere of radius a centered at the origin.
(c) Show that $\text{div}(\vec{F}) = 0$. Does the answer obtained in (*b*) contradict the divergence theorem? Explain.
(d) Let Σ be the surface in the first octant, whose boundary lies in the three coordinate planes (see below). Show that $\int\int_\Sigma \vec{F} \cdot \hat{\mathbf{n}} dS$ is independent of the choice of Σ and calculate its value. (Hint apply the divergence theorem to a suitable portion of the first octant.)