Math 53 - Multivariable Calculus

Homework # 3

July 18th
Due: July 23rd, 2013

Exercise 1.

(a) Describe and sketch the image of the disk of radius 1, $D = u^2 + v^2 \leq 1$, under the transformation $x = au$, $y = bv$, where $a, b \in \mathbb{R}$ and $a \neq 0$, $b \neq 0$.

(b) Use the ideas from part (a) to find the area enclosed by the ellipse $(2x + 5y - 3)^2 + (3x - 7y + 8)^2 = 1$.

Exercise 2.

Evaluate $\displaystyle\int_{\mathbb{R}} e^{-ax^2}\, dx$, where a is any positive real number. (Hint: Let $I = \int_{\mathbb{R}} e^{-ax^2}\, dx$ then write I^2 as a double integral.)

Exercise 3.

Show that a constant force field does zero work on a particle that winds uniformly w times around the ellipse $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$.

Exercise 4.

Compute $\int_0^1 \int_0^1 e^{\max(x^2,y^2)} dydx$ where $\max(x^2, y^2)$ means the larger of the numbers x^2 and y^2.

Exercise 5.

Evaluate $\displaystyle\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ where $\vec{\boldsymbol{F}}(x, y) = \left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \right\rangle$ and C is the parabola $y = 1 + x^2$ from $(-1, 2)$ to $(1, 2)$.