Math 53 - Multivariable Calculus

Homework # 2

July 3rd
Due: July 9th, 2013

Exercise 1. (*a*) *Show that if a particle moves with constant speed then the velocity and acceleration vectors are orthogonal.*
(*b*) *If a particle with mass m moves with position vector* $\vec{\boldsymbol{r}}(t)$ *then its angular momentum is defined as* $\vec{\boldsymbol{l}}(t) := m\vec{\boldsymbol{r}}(t) \times \vec{\boldsymbol{v}}(t)$ *and its torque is defined as* $\vec{\boldsymbol{\tau}}(t) := m\vec{\boldsymbol{r}}(t) \times \vec{\boldsymbol{a}}(t)$. *Show that* $\frac{d}{dt}\vec{\boldsymbol{l}}(t) = \vec{\boldsymbol{\tau}}(t)$ *and deduce that if* $\vec{\boldsymbol{\tau}}(t) = 0$ *for all t, then* $\vec{\boldsymbol{l}}(t)$ *is constant. This is the law of conservation of angular momentum.*

Exercise 2. *Suppose you need to know the equation of a tangent plane to a surface S at the point $p = (2, 1, 3)$. You don't have an equation for S but you do know that the curves*

$$\vec{\boldsymbol{r}}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle, \qquad \vec{\boldsymbol{r}}_2(t) = \langle 1 + u^2, 2u^3 - 1, 2u + 1 \rangle,$$

both lie on S. Find an equation of the tangent plane to S at p.

Exercise 3. *Find the absolute maximum and minimum values of the function given by* $f(x, y) = 4x + 6y - x^2 - y^2$ *on the set* $D = \{(x, y) \mid 0 \leq x \leq 4,\ 0 \leq y \leq 5\}$.

Exercise 4. *Find three positive numbers whose sum is* 12 *and the sum of whose squares is as small as possible.*

Exercise 5. *Suppose that* $x^2y + xz^2 = 5$*, and let* $w = x^3y$*. Express* $\left(\frac{\partial w}{\partial z}\right)_y$ *as a function of* x, y, z *and evaluate it numerically when* $(x, y, z) = (1, 1, 2)$*.*