Math 53 - Multivariable Calculus

Homework # 1

June 27th
Due: July 2nd, 2013

Exercise 1 (Stewart 12.3 # 49). *Use a scalar projection to show that the distance from a point* $P_1 = (x_1, y_1) \in \mathbb{R}^2$ *to the line* $ax + by + c = 0$ *is*

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

Use this formula to find the distance from the point $(-2, 3)$ *to the line* $3x - 4y + 5 = 0$.

Exercise 2 (Stewart 12.3 # 57)**.** *Use that* $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = |\vec{\boldsymbol{A}}||\vec{\boldsymbol{B}}|\cos(\theta)$ *to prove the Cauchy-Schwarz inequality:*

$$|\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}| \leq |\vec{\boldsymbol{A}}||\vec{\boldsymbol{B}}|.$$

Exercise 3 (Stewart 12.4 # 44)**.** *(a) Let P be a point not on the plane that passes through the points Q, R, and S. Show that the distance d from P to the plane is*

$$d = \frac{|(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}) \cdot \vec{\boldsymbol{C}}|}{|\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}|},$$

where $\vec{\boldsymbol{A}} = \vec{\boldsymbol{QR}}$, $\vec{\boldsymbol{B}} = \vec{\boldsymbol{QS}}$, and $\vec{\boldsymbol{C}} = \vec{\boldsymbol{QP}}$. (b) Use the formula from before to find the distance from the point $P(2,1,4)$ to the plane through the points $Q(1,0,0)$, $R(0,2,0)$, and $S(0,0,3)$.

Exercise 4 (The Geometry of a Tetrahedron, Stewart page 794)**.** (*Part 1*) *Let* $\vec{\boldsymbol{V}}_1$, $\vec{\boldsymbol{V}}_2$, $\vec{\boldsymbol{V}}_3$, *and* $\vec{\boldsymbol{V}}_4$ *be vectors with lengths equal to the areas of the faces opposite the vertices* P, Q, R, *and* S, *respectively, and directions perpendicular to the respective faces an pointing outward. Show that*

$$\sum_{i=1}^{4} \vec{\boldsymbol{V}}_i = \vec{\mathbf{0}}.$$

(*Part 2*) *Suppose the tetrahedron in the figure* (*see Stewart page 794*) *has a trirectangular vertex* S. *Let* A, B, *and* C *be the areas of the three faces that meet at* S, *and let* D *be the area of the opposite face* PQR. *Using the results of part 1, or otherwise, show that*

$$D^2 = A^2 + B^2 + C^2.$$

Exercise 5 (Stewart 12.5 #13, #52)**.** (*a*) *Is the line through* $(-4,-6,1)$ *and* $(-2,0,-3)$ *perpendicular to the line through* $(-3,2,0)$ *and* $(5,1,4)$*?*
(*b*) *Determine whether the planes*

$$2x - 3y + 4z = 5, \qquad x + 6y + 4z = 3,$$

are parallel, perpendicular, or neither. If neither, find the angle between them.