Math 53 - Multivariable Calculus

Midterm # 2

August 4th, 2011

Exercise 1.

(*a*) (5 *points*) *Evalute the integral*

$$\int_0^1 \int_0^1 e^{max\{x^2,y^2\}} dydx,$$

where $max\{x^2, y^2\}$ *means the larger of the numbers* x^2 *and* y^2.

(*b*) (5 *points*) *Find the volume of the solid* E *that lies above the cone* $z = \sqrt{x^2+y^2}$ *and below the sphere* $x^2+y^2+z^2=1$.

Exercise 2. (5 *points*) *How many functions* $f : \mathbb{R}^2 \to \mathbb{R}$ *are there whose partial derivatives are* $f_x(x, y) = x + 4y$ *and* $f_y(x, y) = 3x - y$*. Prove your response.*

Exercise 3. (10 *points*) *Show that a constant force field does zero work on a particle that moves once uniformly around the circle* $x^2 + y^2 = 1$. *Is this also ture for a force field* $\vec{\boldsymbol{F}}(\vec{\boldsymbol{x}}) = k\vec{\boldsymbol{x}}$, *where* $k \in \mathbb{R}$ *and* $\vec{\boldsymbol{x}} = \langle x, y \rangle$.

Exercise 4. (15 *points*) *Use the divergence theorem to compute the flux of* $\vec{\boldsymbol{F}} = \langle z^2 x, \frac{1}{3}y^3 + \tan(z), x^2 z + y^2 \rangle$ *through the top half of the sphere* S*,* $x^2 + y^2 + z^2 = 1$.

Exercise 5.

(a) (5 *points*) *Show that the vector field*

$$\vec{\boldsymbol{F}} = \langle e^x yz, e^x z + 2yz, e^x y + y^2 + 1\rangle$$

is conservative.

(b) (10 *points*) *Find a potential for* $\vec{\boldsymbol{F}}$.

(c) (5 *points*) *Prove whether the vector field* $\vec{\boldsymbol{G}} = \langle y, x, y\rangle$ *is conservative or not. If so, find a potential for* $\vec{\boldsymbol{G}}$.

Exercise 6. (10 *points*) *Suppose a particle moves along line segments from the origin to the points* $(1, 0, 0)$, $(1, 2, 1)$, $(0, 2, 1)$, *and back to the origin under the influence of the force field* $\vec{\boldsymbol{F}} = \langle z^2, 2xy, 4y^2 \rangle$. *Find the work done.*

Exercise 7. (15 *points*) *Let* M *be the solid cylinder* $x^2 + y^2 \leq 1$, $0 \leq z \leq 1$ *and let* $\vec{\boldsymbol{F}} = \langle xy, yz, zx \rangle$. *Verify that the divergence theorem is true for* $\vec{\boldsymbol{F}}$ *on the region* M.

Exercise 8. (15 *points*) *Let D be the upper hemisphere of the unit sphere $x^2 + y^2 + z^2 = 1$. Use Stokes' theorem to evaluate*

$$\int\int_D \left(x^3 e^y \hat{\boldsymbol{\imath}} - 3x^2 e^y \hat{\boldsymbol{\jmath}}\right) \cdot \hat{\boldsymbol{n}}\, dS,$$

where $\hat{\boldsymbol{n}}$ is the upward-pointing normal vector.