Math 53 - Multivariable Calculus

Midterm # 1

July 8th, 2011

Exercise 1. *Argue why or why not the following three planes intersect at a unique point:*

(*a*) (5 *points*)

$$x + 2y = 0, \quad 3y + 5z = 0, \quad 3y = 0.$$

(*b*) (5 *points*)

$$x + 2y + 3z = 0, \quad 2y + 1z = 0, \quad 2x + 4y + 6z = 0.$$

Exercise 2.

(a) (5 points) *Let* $\vec{\boldsymbol{R}} = x(t)\hat{\boldsymbol{\imath}} + y(t)\hat{\boldsymbol{\jmath}} + z(t)\hat{\boldsymbol{k}}$ *be the position vector of some path. Give a simple intrinsic formula for* $\frac{d}{dt}\left(\vec{\boldsymbol{R}} \cdot \vec{\boldsymbol{R}}\right)$ *in vector notation (i.e., not using coordinates).*

(b) (5 points) *Show that if* $\vec{\boldsymbol{R}}$ *has constant length, then* $\vec{\boldsymbol{R}}$ *and* $\vec{\boldsymbol{V}} = \frac{d}{dt}\vec{\boldsymbol{R}}$ *are perpendicular.*

Exercise 3.

(*a*) (10 *points*) *Find the area of the space triangle with vertices* $P_0 = (2,1,0)$, $P_1 = (1,0,1)$, $P_2 = (2,-1,1)$.

(*b*) (10 *points*) *Find the equation of the plane containing the three points* P_0, P_1, P_2.

(*c*) (10 *points*) *Find the intersection of this plane with the line parallel to the vector* $\vec{\boldsymbol{V}} = \langle 1,1,1 \rangle$ *and passing through the point* $S = (-1,0,0)$.

Exercise 4. (10 *points*) *Find the equation of the tangent plane to the surface* $x^3y + z^2 = 3$ *at the point* $(-1, 1, 2)$.

Exercise 5. *Let* $f(x, y) = xy - x^4$

(a) (5 *points*) *Find the gradient of* f *at* $P = (1, 1)$

(b) (10 *points*) *Give an approximate formula telling how small changes* $\triangle x$ *and* $\triangle y$ *produce a small change* $\triangle w$ *in the value* $w = f(x, y)$ *at the point* $(x, y) = (1, 1)$

Exercise 6.

(*a*) (5 *points*) *Suppose* $(1,1)$ *is a critical point of a function* f *with continuous second derivatives. What can you say about* f *given that* $f_{xx}(1,1) = 4$, $f_{xy}(1,1) = 1$, *and* $f_{yy}(1,1) = 2$.

(*b*) (10 *points*) *Find the critical points of* $w = -3x^2 - 4xy - y^2 - 12y + 16x$ *and state what type each critical point is.*

(*c*) (10 *points*) *Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint:*

$$f(x,y) = x^2y; \qquad x^2 + 2y^2 = 6.$$

BONUS

(*a*) (2.5 points) *Suppose the motion of a point P is given by the position vector $\vec{\boldsymbol{R}} = 3\cos(t)\hat{\boldsymbol{\imath}} + 3\sin(t)\hat{\boldsymbol{\jmath}} + t\hat{\boldsymbol{k}}$. Compute the velocity and the speed of P.*

(*b*) (2.5 points) *Sketch the graph the function $f(x, y) = y^2 + 1$.*

(*c*) (2.5 points) *Let $u = y/x$, $v = x^2 + y^2$, and $w = w(u, v)$. Express the partial derivatives w_x and w_y in terms of w_u and w_v.*