Math 53 - Multivariable Calculus

Take Home Assignment # 3

July 13th, 2011

Exercise 1. *Find the image of the disk* $D = \left\{u^2 + v^2 \leq 1 \mid u, v \in \mathbb{R}\right\}$ *under the transformation* $x = au$, $y = bv$. *Here* $a \neq 0$ *and* $b \neq 0$.

Exercise 2. *Evaluate*

$$\int\int_R \sin(9x^2 + 4y^2)dA,$$

where R is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$.

Exercise 3. *Evaluate*

$$\int\int_R e^{-x^2-y^2}\,dxdy,$$

where $R = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$.

Exercise 4. *Given the fact that* $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$, *evaluate*

$$I = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\left(x^2+(y-x)^2+y^2\right)} dxdy.$$

Exercise 5. *Consider the vector field* $\vec{\boldsymbol{F}}(x,y) = \frac{x}{\sqrt{x^2+y^2}}\hat{\boldsymbol{\imath}} + \frac{y}{\sqrt{x^2+y^2}}\hat{\boldsymbol{\jmath}}$. *Evaluate the following line integral*

$$\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}},$$

where C *is the parabola* $y = 1 + x^2$ *from* $(-1, 2)$ *to* $(1, 2)$.

Exercise 6. *The base of a circular fence with radius* $10m$ *is given by* $x = 10\cos(t)$, $y = 10\sin(t)$. *The height of the fence at position* (x, y) *is given by the function* $h(x, y) = 4 + 0.01(x^2 - y^2)$, *so the height varies from* $3m$ *to* $5m$. *Suppose that* $1L$ *of paint covers* $100m^2$. *Sketch the fence and determine how much paint you will need to paint both sides of the fence.*