Math 53 - Multivariable Calculus

Take Home Assignment # 2

June 30th, 2011

Exercise 1. *Suppose you have two particles, one is traveling along the space curve given by* $\vec{\boldsymbol{r}}_1(t) = \langle t, t^2, t^3 \rangle$ *while the other particle is traveling along* $\vec{\boldsymbol{r}}_2(t) = \langle 1 + 2t, 1 + 6t, 1 + 14t \rangle$*. Do the particles collide? Do thier paths intersect?*

Exercise 2. *Show that the curvature of a plane curve is* $\kappa = |d\phi/ds|$*, where* ϕ *is the angle between* $\vec{\boldsymbol{T}}$ *and* $\hat{\boldsymbol{\imath}}$*; that is,* ϕ *is the angle of inclination of the tangent line.*

Exercise 3.

(a) *Show that if a particle moves with constant speed, then the velocity and acceleration vectors are orthogonal.*

(b) *A batter hits a baseball 3 ft above the ground toward the center field fence, which is 10 ft high and 400 ft from home plate. The ball leaves the bat with speed 115 ft/s at an angle 50° above the horizontal. Is it a home run (i.e., does the ball clear the fence)?*

(c) *If a particle with mass m moves with position vector $\vec{\boldsymbol{r}}(t)$, then its angular momentum is defined as*

$$\vec{\boldsymbol{L}}(t) := m\vec{\boldsymbol{r}}(t) \times \vec{\boldsymbol{v}}(t),$$

and its torque is defined as

$$\vec{\boldsymbol{\tau}}(t) := m\vec{\boldsymbol{r}}(t) \times \vec{\boldsymbol{a}}(t).$$

Show that $\vec{\boldsymbol{L}}'(t) = \vec{\boldsymbol{\tau}}(t)$ and deduce that if $\vec{\boldsymbol{\tau}}(t) = \vec{\mathbf{0}}$ for all t, then $\vec{\boldsymbol{L}}(t)$ is constant. This is the law of conservation of angular momentum.

Exercise 4. *If a, b, c are sides of a triangle and A, B, C are the opposite angles, find $\partial A/\partial a, \partial A/\partial b, \partial A/\partial c$. Hint: easiest to do by implicite differentiation of the Law of Cosines (just google if you don't know the Law of Cosines).*

Exercise 5. *Suppose you need to know an equation of the tangent plane to a surface S at the point $P = (2, 1, 3)$. You don't have an equation for S but you know that the curves*

$$\vec{\boldsymbol{r}}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle$$
$$\vec{\boldsymbol{r}}_2(u) = \langle 1 + u^2, 2u^3 - 1, 2u + 1 \rangle$$

both lie in S. Find an equation of the tangent plane at P.

Exercise 6.

(*a*) *Find the absolute maximum and minimum values of* $f(x, y) = 4x + 6y - x^2 - y^2$ *on the set* $D = \{(x, y) \mid 0 \leq x \leq 4, 0 \leq y \leq 5\}$.

(*b*) *Find three positive numbers whose sum is 12 and the sum of whose squares is as small as possible.*

(*c*) *Find the dimensions of the rectangular box with largest volume if the total surface area is given as* $64\ cm^2$