Math 53 - Multivariable Calculus

Take Home Assignment # 1

June 23rd, 2011

Exercise 1.

Let V be a vector space over $\mathbb{R}$ (think of $\mathbb{R}^n$), an inner product on V is a map $\langle \cdot, \cdot \rangle \longrightarrow \mathbb{R}$ such that, for all $x, y, z \in V$ and $\alpha, \beta \in \mathbb{R}$,

(i) $\langle x, y \rangle = \langle y, x \rangle$,

(ii) $\langle \alpha x + \beta y, z \rangle = \alpha \langle x, z \rangle + \beta \langle y, z \rangle$,

(iii) $\langle x, x \rangle \geq 0$ with equality only for $x = 0$.

One can check that the 'dot' product, $\langle x, z \rangle = x \cdot z = \sum_{i=1}^{n} \alpha_i \beta_i$, introduced in class is AN inner product, but not the only one. In particular, show that $\langle \cdot, \cdot \rangle \longrightarrow \mathbb{R}$, defined by

$$\langle f, g \rangle = \int_0^1 f(x) g(x) dx,$$

is an inner product on $C(\mathbb{R})$, the vector space of all continuous real-valued functions.

Exercise 2. *Find a unit vector that is perpendicular to both* $\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}$ *and* $\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{k}}$.

Exercise 3. *Use that* $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = |\vec{\boldsymbol{A}}||\vec{\boldsymbol{B}}| \cos(\theta)$ *to prove the Cauchy-Schwarz inequality* $|\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}| \leq |\vec{\boldsymbol{A}}||\vec{\boldsymbol{B}}|$.

Exercise 4. *Find the volume of the parallelpiped determined by the vectors* $\vec{\boldsymbol{A}} = \hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} - \hat{\boldsymbol{k}}$, $\vec{\boldsymbol{B}} = \hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}} + \hat{\boldsymbol{k}}$, *and* $\vec{\boldsymbol{C}} = -\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} + \hat{\boldsymbol{k}}$.

Exercise 5. *Is the line through* $(-4, -6, 1)$ *and* $(-2, 0, -3)$ *perpendicular to the line through* $(-3, 2, 0)$ *and* $(5, 1, 4)$*?*

Exercise 6. *Determine whether the planes determined by*

$$2x - 3y + 4z = 5, \qquad x + 6y + 4z = 3$$

are parallel, perpendicular, or neither. If neither, find the angle between them.