Math 53 - Multivariable Calculus

Take Home Assignment # 1

June 23rd, 2011

Exercise 1.

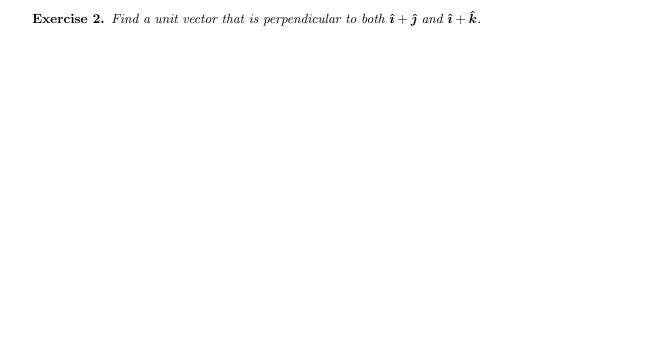
Let V be a vector space over \mathbb{R} (think of \mathbb{R}^n), an inner product on V is a map $\langle \cdot, \cdot \rangle \longrightarrow \mathbb{R}$ such that, for all $x, y, z \in V$ and $\alpha, \beta \in \mathbb{R}$,

- $(i) \ \langle x, y \rangle = \langle y, x \rangle,$
- (ii) $\langle \alpha x + \beta y, z \rangle = \alpha \langle x, z \rangle + \beta \langle y, z \rangle$,
- $(iii) \ \langle x,x\rangle \geq 0 \text{ with equality only for } x=0.$

One can check that the 'dot' product, $\langle x, z \rangle = x \cdot z = \sum_{i=1}^{n} \alpha_i \beta_i$, introduced in class is AN inner product, but not the only one. In particular, show that $\langle \cdot, \cdot \rangle \longrightarrow \mathbb{R}$, defined by

$$\langle f, g \rangle = \int_0^1 f(x)g(x)dx,$$

is an inner product on $C(\mathbb{R})$, the vector space of all continuous real-valued functions.



Exercise 3. Use that $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos(\theta)$ to prove the Cauchy-Schwarz inequality $|\vec{A} \cdot \vec{B}| \leq |\vec{A}| |\vec{B}|$.



Exercise 6. Determine whether the planes determined by

$$2x - 3y + 4z = 5,$$
 $x + 6y + 4z = 3$

are parallel, perpendicular, or neither. If neither, find the angle between them.