Math 53 - Multivariable Calculus

Final

August 12th, 2011

Exercise 1. *Let* $P = (1, 1, 1)$, $Q = (0, 3, 1)$ *and* $R = (0, 1, 4)$.

(*a*) (10 *points*) *Find the area of the triangle* PQR.

(*b*) (5 *points*) *Find the plane through* P, Q, *and* R, *expressed in the form* $ax + by + cz = d$.

(*c*) (5 *points*) *Is the line through* $(1, 2, 3)$ *and* $(2, 2, 0)$ *parallel to the plane in part* (*b*)*? Explain why or why not.*

Exercise 2. (10 *points*) *Show that the curve with parameteric equations* $x = t\cos(t)$, $y = t\sin(t)$, $z = t$ *lies on the cone* $z^2 = x^2 + y^2$, *and use this to help sketch the curve.*

Exercise 3. (10 *points*) *Find the linear approximation of the function* $f(x, y) = \sqrt{20 - x^2 - 7y^2}$ *at* $(2, 1)$ *and use it to approximate* $f(1.95, 1.08)$.

Exercise 4.

(a) (5 *points*) *Evaluate* $\int_c \vec{F} \cdot d\vec{r}$ *where* $\vec{F} = \langle e^y, xe^y + e^z, ye^z \rangle$ *and* c *is the line segment from* $(0,2,0)$ *to* $(4,0,3)$.

(b) (5 *points*) *Show that there is no vector field* $\vec{F}$ *such that* $\nabla \times \vec{F} = \langle 2x, 3yz, xz^2 \rangle$.

(c) (5 *points*) *If* c *is any piecewise-smooth simple closed plane curve and* f *and* g *are differentiable functions, show that* $\int_c f dx + g dy = 0$, *if* $f = f(x)$ *and* $g = g(y)$.

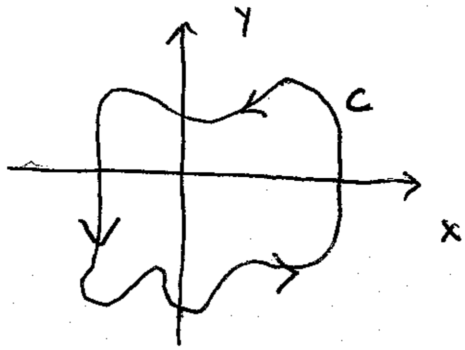
Exercise 5.

(*a*) (5 *points*) *Evaluate the surface integral* $\int\int_S \vec{F}\cdot\hat{n}dS$*, where* $\vec{F} = \langle xz, -2y, 3x\rangle$ *and* S *is the sphere* $x^2+y^2+z^2=4$ *with outward orientation.*

(*b*) (10 *points*) *Let*

$$\vec{F} = \frac{(2x^3+2xy^2-2y)\hat{\imath} + (2y^3+2x^2y+2x)\hat{\jmath}}{x^2+y^2}.$$

Evaluate $\int_c \vec{F}\cdot d\vec{r}$ *where c is shown below:*



Exercise 6. (15 *points*) *The plane* $x + y + 2z = 2$ *intersects the paraboloid* $z = x^2 + y^2$ *in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin.*

Exercise 7.

(a) (5 *points*) *If* f *is a harmonic function, that is* $(\nabla\cdot\nabla)f = 0$, *show that the line integral* $\int_c f_y dx - f_x dy$ *is path independent in any simply connected region* D.

(b) (5 *points*) *Use Stokes' theorem to evaluate* $\int_c \vec{F}\cdot d\vec{r}$, *where* $\vec{F} = \langle xy, yz, zx\rangle$ *and* c *is the triangle with vertices* $(1,0,0)$, $(0,1,0)$ *and* $(0,0,1)$, *oriented counter-clockwise as viewed from above.*

(c) (5 *points*) *Let* $\vec{F} = \langle 3x^2yz - 3y, x^3z - 3x, x^3y + 2z\rangle$. *Evaluate* $\int_c \vec{F}\cdot d\vec{r}$, *where* c *is the curve drawn below:*