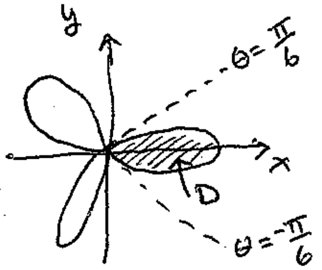
Math 53 - Multivariable Calculus

Quiz # 8

Solns

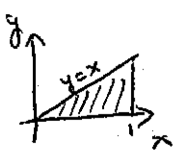
March 16th, 2012

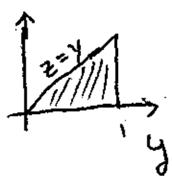
Exercise 1. *Use a double integral to find the area of the region which is ONE loop of the rose $r = \cos(3\theta)$.*



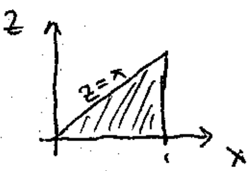
$$A(D) = \iint_D dA = \int_{-\pi/6}^{\pi/6}\int_0^{\cos(3\theta)} = \int_{-\pi/6}^{\pi/6} \frac{1}{2}\cos^2(3\theta)\,d\theta = \frac{\pi}{12}$$

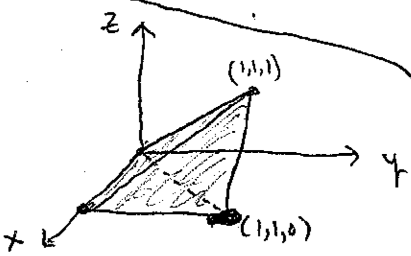
Exercise 2. *Consider the integral $\int_0^1 \int_y^1 \int_0^y dz\ dx\ dy$, figure out the limits for $\int\int\int dy\ dz\ dx$ and $\int\int\int dx\ dy\ dz$.*



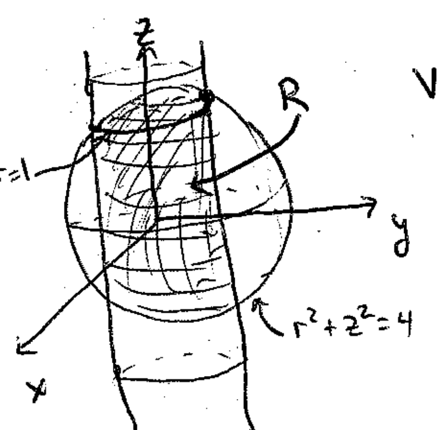


$$\int_0^1\int_0^x\int_z^x dy\,dz\,dx\ , \quad \int_0^1\int_z^1\int_y^1 dx\,dy\,dz$$





Exercise 3. *Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.*



$$\mathrm{Vol}(R) = \int_0^{2\pi}\int_0^1\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta = \frac{4}{3}\pi\left(8-3\sqrt{3}\right)$$

Exercise 4 (Bonus 2pt.). *Evaluate $I = \int_{-\infty}^{\infty} e^{-x^2} dx$. (Hint, square I and then convert to polar coordinates.)*

$$I^2 = \iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\,dx\,dy = \int_0^{2\pi}\int_0^{\infty} e^{-r^2}\,r\,dr\,d\theta = \int_0^{2\pi} -\frac{1}{2}\left(e^{-\infty} - e^0\right)d\theta$$

$$= \int_0^{2\pi} \frac{1}{2}\,d\theta = \pi \quad \Rightarrow \quad I = \sqrt{\pi}$$