Math 53 - Multivariable Calculus

Quiz # 7 — Solns

March 9th, 2012

Exercise 1. *Suppose* $(1,1)$ *is a critical point of a function* f *with continuous second derivatives. What can you say about* f *given that* $f_{xx}(1,1) = -4$, $f_{yy}(1,1) = 1$, *and* $f_{xy}(1,1) = -5$.

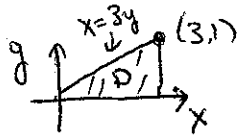
$f_{xx}(1,1)\, f_{yy}(1,1) - \left(f_{xy}(1,1)\right)^2 = (-4)(1) - (-5)^2 = -4 - 25 = -29 < 0$

$\Rightarrow$ f has a saddle point at the point $(1,1) \in \mathbb{R}^2$.

Exercise 2. *Find and classify ALL the critical points of* $f(x,y) = e^x \cos(y)$.

$f_x = e^x \cos(y)$ and $f_y = e^x \sin(y)$. So $f_x = 0$ when $y = \frac{\pi}{2} + n\pi$. However $\sin(\frac{\pi}{2} + n\pi) \neq 0 \Rightarrow e^x \sin(\frac{\pi}{2} + n\pi) \neq 0 \Rightarrow$ no local critical points.

Exercise 3. *Evaluate* $\int_0^1 \int_{3y}^3 e^{x^2}\, dx dy$.



Let's first switch the order of integration, $\int_0^1 \int_{3y}^3 e^{x^2}\, dx dy =$

$$\int_0^3 \int_0^{x/3} e^{x^2}\, dy\, dx = \int_0^3 \left[e^{x^2} y \Big|_0^{x/3} \right] dx = \int_0^3 \left(\frac{x}{3}\right) e^{x^2}\, dx = \frac{1}{6} e^{x^2} \Big|_0^3 =$$

$$= \frac{e^9 - 1}{6}$$