# Math 53 - Multivariable Calculus

Quiz # 6

Solns

March 2nd, 2012

**Exercise 1.** *Show that if $z(x,y) = f(x-y)$ then $z$ satisfies the partial differential equation given by $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.*

Let $u = x - y$ then $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} = \frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} = -\frac{\partial z}{\partial u}$. So,

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} - \frac{\partial z}{\partial u} = 0.$$

**Exercise 2.** *Let $u = y/x$, $v = x^2 + y^2$, $w = w(u,v)$. Express the partial derivatives $w_x$ and $w_y$ in terms of $w_u$ and $w_v$ ( and $x$ and $y$ ).*

$w = w(u(x,y), v(x,y))$. So, by the chain rule:

$$w_x = w_u u_x + w_v v_x = \frac{-y}{x^2} w_u + 2x w_v$$

$$w_y = w_u u_y + w_v v_y = \frac{1}{x} w_u + 2y w_v$$

**Exercise 3.** *Let $f(x,y) = x^2y^2 - x$. Find the gradient $\nabla f$ at $(2,1)$. Write the equation for the tangent plane to the graph of $f$ at $(2,1,2)$. Now, use a linear approximation to find the approximate value of $f(1.9, 1.1)$. Finally, find the directional derivative of $f$ at $(2,1)$ in the direction $\langle -1, 1\rangle$.*

$\nabla f = \langle 2xy^2 - 1, 2x^2y \rangle \Rightarrow \nabla f \big|_{(2,1)} = \langle 3, 8 \rangle$. The eq$^{\underline{n}}$ of the tangent plane is $(z-2) = 3(x-2) + 8(y-1)$ or $z = 3x + 8y - 12$. Further, $\Delta x = 2 - 1.9 = -1/10$ $\Delta y = 1 - 1.1 = 1/10 \Rightarrow \Delta z \approx -\frac{3}{10} + \frac{8}{10} \Rightarrow z \approx 2 + \frac{5}{10} = 2.5$. Finally,

$$\frac{df}{ds}\Big|_{\hat{u}} = \nabla f \cdot \hat{u} = \langle 3, 8 \rangle \cdot \frac{\langle -1, 1 \rangle}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$