Math 53 - Multivariable Calculus

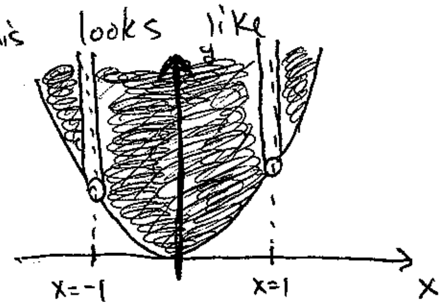
Quiz # 5

February 24th, 2012

Solns

Exercise 1. *Find and sketch the DOMAIN of the function f when $f(x,y) = \frac{\sqrt{y-x^2}}{1-x^2}$.*

The domain is $\{(x,y) \mid y-x^2 \geq 0 \text{ and } 1-x^2 \neq 0\}$. Now $y-x^2 \geq 0 \iff y \geq x^2$ and $1-x^2 \neq 0 \iff x \neq \pm 1$. This looks like



Exercise 2. *Find the limit or prove that it does not exist for* $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$.

We claim $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0$. We'll use the squeeze theorem. So, note

$$0 \leq \left|\frac{xy}{\sqrt{x^2+y^2}}\right| = |x| \underbrace{\left|\frac{y}{\sqrt{x^2+y^2}}\right|}_{\leq 1} \leq |x|.$$

Since $|x| \to 0$ as $(x,y) \to (0,0)$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0.$$

Exercise 3. *State, with proof, the number of functions f that have partial derivatives $f_x(x,y) = x + 4y$ and $f_y(x,y) = 3x - y$. (Hint: your reasoning should NOT involve solving any differential equations.)*

There is NO $f^{\underline{n}}$ that has $f_x = x + 4y$ and $f_y = 3x - y$. Indeed, suppose such a $f^{\underline{n}}$ existed. Then $f_{xy} = 4$ while $f_{yx} = 3$. This violates Clairaut's theorem.