Math 53 - Multivariable Calculus

Solns

Quiz # 4

February 10th, 2012

Exercise 1. *Find two UNIT vectors orthogonal to both $\langle 1, -1, 1\rangle$ and $\langle 0, 4, 4\rangle$.*

If $\vec{A} = \langle 1,-1,1\rangle \times \langle 0,4,4\rangle = \langle -8,-4,4\rangle \Rightarrow \vec{A}$ and $\vec{B} = -\vec{A}$ are both perpendicular to $\langle 1,-1,1\rangle$ and $\langle 0,4,4\rangle$. So, take $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \left\langle -\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right\rangle$ and $\hat{B} = \left\langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right\rangle$.

Exercise 2. *Suppose that $\vec{A}$ is not the zero vector, $\vec{A} \neq \vec{0}$. If $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C}$, does it follow that $\vec{B} = \vec{C}$? If $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$, does it follow that $\vec{B} = \vec{C}$? If $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C}$ and $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$, does it follow that $\vec{B} = \vec{C}$?*

No, $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C} \Rightarrow \vec{A}\cdot(\vec{B}-\vec{C}) = 0 \Rightarrow \vec{A} \perp \vec{B}-\vec{C}$ which can happen for e.g. $\vec{A} = \langle 1,1,1\rangle$ $\vec{B} = \langle 1,0,0\rangle$ and $\vec{C} = \langle 0,1,0\rangle$. The next is also No. Indeed, if $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$ $\Rightarrow \vec{A}\times(\vec{B}-\vec{C}) = 0 \Rightarrow \vec{A} \parallel \vec{B}-\vec{C}$ and this can also happen even if $\vec{B} \neq \vec{C}$.

Finally, we claim that the two previous conditions together imply that $\vec{B} = \vec{C}$. Indeed if $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C} \Rightarrow \vec{A} \perp \vec{B}-\vec{C}$ and if $\vec{A}\times\vec{B} = \vec{A}\times\vec{C} \Rightarrow \vec{A} \parallel \vec{B}-\vec{C}$ $\Rightarrow \vec{A} \perp$ and $\parallel \vec{B}-\vec{C} \Rightarrow$ either $\vec{A} = \vec{0}$ (which cannot be) or $\vec{B}-\vec{C} = \vec{0}$ $\Rightarrow \vec{B}-\vec{C} = \vec{0} \Rightarrow \vec{B} = \vec{C}$.

Exercise 3. *Find the equation of the plane through the origin and parallel to the plane $2x - y + 3z = 1$.*

Since the planes are parallel $\Rightarrow$ they have same normal vector. $\Rightarrow$ the eq$\underline{n}$ will be of the form $2x - y + 3z = C$. To find C, plug in any point on the plane, for e.g. $2(0) - (0) + 3(0) = 0 \Rightarrow C = 0 \Rightarrow$ eq$\underline{n}$ of the plane is $2x - y + 3z = 0$.