Math 53 - Multivariable Calculus

Solns

Quiz # 3

February 3rd, 2012

Exercise 1. *Suppose that $\vec{r} = \langle x, y, z\rangle$ and $\vec{r}_0 = \langle x_0, y_0, z_0\rangle$, describe the set of all points (x, y, z) such that $|\vec{r} - \vec{r}_0| = 1$.*

$|\vec{r}-\vec{r}_0| = 1 \Leftrightarrow \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} = 1 \Leftrightarrow$

$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = 1$, this is the equation of a sphere with radius 1 centered at the point (x_0, y_0, z_0).

Exercise 2. *Determine whether the two vectors $\vec{A} = \langle -5, 3, 7\rangle$ and $\vec{B} = \langle 6, -8, 2\rangle$ are perpendicular, parallel, or neither.*

Well, clearly $\nexists\ \lambda \in \mathbb{R}$ such that $\vec{A} = \lambda\vec{B} \Rightarrow \vec{A}$ and $\vec{B}$ are not parallel. Next, $\vec{A}\cdot\vec{B} = (-5)(6) + (3)(-8) + (7)(2) = -30 - 24 + 14 \neq 0 \Rightarrow \vec{A}$ and $\vec{B}$ are not orthogonal.

Exercise 3. *Find a UNIT vector that is orthogonal to both $\hat{\imath} + \hat{\jmath}$ and $\hat{\imath} + \hat{k}$. (Hint: Assume that $\vec{A} = \langle a_1, a_2, a_3\rangle$ is such a vector and use the orthogonality to determine the value of $\vec{A}$'s coefficients, a_i.)*

Suppose $\vec{A} = \langle a_1, a_2, a_3\rangle$ is such a vector $\Rightarrow (\hat{\imath}+\hat{\jmath})\cdot\vec{A} = 0$ implies $a_1 + a_2 = 0$ and $(\hat{\imath}+\hat{k})\cdot\vec{A} = 0$ implies $a_1 + a_3 = 0$. Solving these two eq$^{\underline{ns}}$ simultaneously we have $a_1 = -a_2 = -a_3$. Now, if $\vec{A}$ is a unit vector then $|\vec{A}| = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{a_1^2 + a_2^2 + a_3^2} = 1 \Leftrightarrow a_1^2 + a_2^2 + a_3^2 = 1 \Leftrightarrow 3a_1^2 = 1$ (since $a_1 = -a_2 = -a_3$) $\Leftrightarrow a_1 = \pm\frac{1}{\sqrt{3}}$. So, let's take $a_1 = \frac{1}{\sqrt{3}}$, then we get $\vec{A} = \langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\rangle$, the desired vector.