

# Math 53 - Multivariable Calculus

Quiz # 1

January 25th, 2012

Solns

**Exercise 1.** Find a polar equation for the curve represented in Cartesian coordinates by  $x^2 + y^2 = 2cx$ , where here  $c \in \mathbb{R}$ .

$x^2 + y^2 = 2cx \iff r^2 = 2c r \cos(\theta) \iff r^2 - 2c r \cos(\theta) = 0 \iff r(r - 2c \cos(\theta)) = 0$   
 $\Rightarrow r = 0$  or  $r = 2c \cos(\theta)$ . However, when  $\theta = \frac{\pi}{2}$  (or any integral multiple of  $\frac{\pi}{2}$ )  $2c \cos(\theta) = 0$ . Hence, the curve is represented by the single equation  $r = 2c \cos(\theta)$ .

**Exercise 2.** Show that the polar equation  $r = a \sin(\theta) + b \cos(\theta)$ , where  $ab \neq 0$ , represents a circle, and find its center and radius.

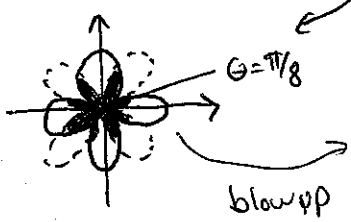
$r = a \sin(\theta) + b \cos(\theta) \Rightarrow r^2 = a r \sin(\theta) + b r \cos(\theta) \Rightarrow x^2 + y^2 = ay + bx \Rightarrow$   
 $x^2 - bx + y^2 - ay = 0 \Rightarrow x^2 - bx + (\frac{1}{2}b)^2 - (\frac{1}{2}b)^2 + (\frac{1}{2}a)^2 - (\frac{1}{2}a)^2 = 0$   
 $\Rightarrow x^2 - bx + (\frac{1}{2}b)^2 + y^2 - ay + (\frac{1}{2}a)^2 = (\frac{1}{2}b)^2 + (\frac{1}{2}a)^2 \Rightarrow (x - \frac{1}{2}b)^2 + (y - \frac{1}{2}a)^2 = \frac{1}{4}(a^2 + b^2)$   
 This is the equation of a circle with center  $(\frac{1}{2}b, \frac{1}{2}a)$  and radius  $\frac{1}{2}\sqrt{a^2 + b^2}$ .

**Exercise 3.** find the area of the region that lies inside the curves  $r = \sin(2\theta)$  and  $r = \cos(2\theta)$ . (Hint:  $2 \sin^2(2\theta) = 1 - \cos(4\theta)$ )

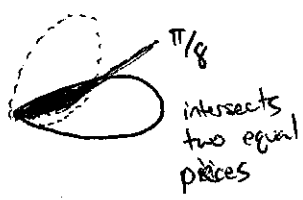
$$\sin(2\theta) = \cos(2\theta) \Rightarrow \frac{\sin(2\theta)}{\cos(2\theta)} = 1 \Rightarrow \tan(2\theta) = 1 \Rightarrow 2\theta = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{8}$$

$$\Rightarrow A = \int_a^b \frac{1}{2} (f(\theta))^2 d\theta = 2 \cdot 8 \int_0^{\pi/8} \frac{1}{2} \sin^2(2\theta) d\theta = 8 \int_0^{\pi/8} \frac{1}{2} (1 - \cos(4\theta)) d\theta = \frac{\pi}{2} - 1$$

by symmetry



blowup



intersects two equal pieces

better picture:

