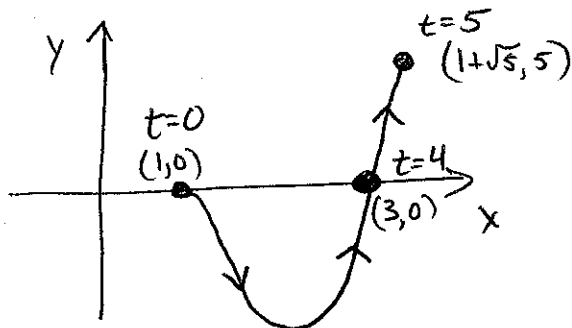# Math 53 - Multivariable Calculus

Quiz # 1

January 20th, 2012

Solns

Exercise 1. *Sketch the curve given by* $x = 1 + \sqrt{t}$, $y = t^2 - 4t$, *where* $0 \le t \le 5$, *and indicate the orientation of the curve (i.e., indicate with an arrow the direction in which the curve is traced as t increases).*

Exercise 2. *Describe the motion of a particle with position* $(x(t), y(t))$, *where* $x(t) = 5\sin(t)$, $y(t) = 2\cos(t)$ *and* $-\pi \le t \le 5\pi$.

We have : $x = 5\sin(t)$, $y = 2\cos(t) \Longleftrightarrow \sin(t) = \frac{x}{5}$, $\cos(t) = \frac{y}{2}$ $\Rightarrow \left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

$\Rightarrow$ particle moves on an ellipse centered at $(0,0)$. When $t = -\pi$ the particle is at $(0,-2)$. As t ranges over $[-\pi, 5\pi]$ the particle starts at $(0,-2)$ and moves clockwise around the ellipse three times.

Exercise 3. *Find the parametric equations for the path of a particle that moves once ~~counter~~clockwise around the circle* $x^2 + (y-1)^2 = 4$ *starting at* $(2,1)$.

The circle $x^2 + (y-1)^2 = 4$ has center $(0,1)$ and radius $r = 2$.
We can parameterize it as $x = 2\cos(t)$, $y = 1 + 2\sin(t)$, $0 \le t \le 2\pi$.
Now, since we want a clockwise orientation, we change to:

$$x = 2\cos(t), \quad y = 1 - 2\sin(t), \quad 0 \le t \le 2\pi$$