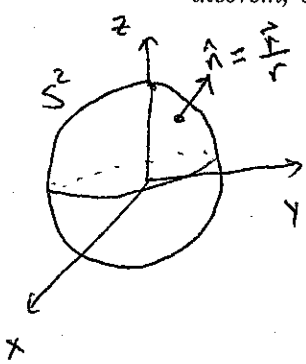
Math 53 - Multivariable Calculus

Quiz # 10 Solns

April 20th, 2012

Exercise 1. *Suppose there is an elementary particle, of charge g, located at the origin that generates a force field given by $\vec{A}(\vec{r}) = g\frac{\vec{r}}{r^3}$, where r denotes the magnitude of $\vec{r}$. Use a surface integral to compute the flux of this force through a sphere of radius $r > 0$ centered at the origin (i.e., DON'T use the divergence theorem, explicitely compute the surface integral).*



$$\text{Flux} = \iint_{S^2} \vec{A}\cdot\hat{n}\, dS = \iint_{S^2} g\frac{\vec{r}}{r^3}\cdot\frac{\vec{r}}{r}\, dS$$

$$= g \iint_{S^2} \frac{1}{r^2}\, dS$$

$$= g \int_0^{2\pi}\int_0^{\pi} \frac{1}{r^2} r^2 \sin\varphi \, d\varphi\, d\theta$$

$$= g\, 4\pi$$

Exercise 2. *Let S_1^2 denote the unit sphere, $S_1^2 = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 = 1\}$. Use the divergence theorem to compute the surface integral $\iint_{S_1^2}(x^2+y+z)dS$.*

Let $\vec{F} = \langle x,1,1\rangle$. Then for S_1^2, $\hat{n} = \frac{\langle x,y,z\rangle}{1}$ and $\vec{F}\cdot\hat{n} = x^2+y+z$.

So, $$\iint_{S_1^2} (x^2+y+z)\, dS = \iint_{S_1^2} \vec{F}\cdot\hat{n}\, dS = \iiint_{x^2+y^2+z^2\le 1} \operatorname{div}(\vec{F})\, dV = \iiint_{x^2+y^2+z^2\le 1} dV = \frac{4}{3}\pi$$

Exercise 3. *Compute the flux of $\vec{F}(x,y,z) = \langle xy\sin(z), \cos(xz), y\cos(z)\rangle$ through any positively oriented closed surface S that contains the origin.*

Let S be any positively oriented closed surface containing the origin. Then, by the divergence theorem, $\iint_S \vec{F}\cdot\hat{n}\, dS = \iiint_B \operatorname{div}(\vec{F})\, dV = 0$ since $\operatorname{div}(\vec{F}) = 0$.

where $\partial B = S$