Math 53 - Multivariable Calculus

Quiz # 9

November 3rd, 2011

Solns

Exercise 1. *Evaluate the line integral* $\int_C (x^2y^3 - \sqrt{x})dy$*, where* C *is the arc of the curve* $y = \sqrt{x}$ *from* $(1,1)$ *to* $(4,2)$.

Let x be parameter, then C given by $x = x,\ y = \sqrt{x}\ ,\ 1 \le x \le 4$

$$\Rightarrow \int_C (x^2y^3 - \sqrt{x})\,dy = \int_1^4 \left(x^2 \cdot (\sqrt{x})^3 - \sqrt{x}\right)\frac{1}{2\sqrt{x}}\,dx = \frac{1}{2}\int_1^4 (x^3 - 1)\,dx = \frac{243}{8}$$

Exercise 2. *Evaluate the line integral* $\int_C \vec{F} \cdot d\vec{r}$*, where* $\vec{F} = \sin(x)\hat{\imath} + \cos(y)\hat{\jmath} + xz\hat{k}$ *and* C *is the curve given by the vector function* $\vec{r}(t) = t^3\hat{\imath} - t^2\hat{\jmath} + t\hat{k}$, $0 \le t \le 1$.

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^1 \langle \sin(t), \cos(-t^2), t^4 \rangle \cdot \langle 3t^2, -2t, 1 \rangle\, dt$$

$$\int_C \left(\vec{F}(\vec{r}(t)) \cdot \frac{d\vec{r}}{dt}\right) dt$$

$$= \int_0^1 \left(3t^2 \sin(t^3) - 2t\cos(t^2) + t^4\right) dt = \frac{6}{5} - \cos(1) - \sin(1)$$

Exercise 3. *Recall, the work done by a force field* $\vec{F}$ *on a particle along some trajectory* C *is given by the line integral* $\int_C \vec{F} \cdot d\vec{r}$. *Use this to show that a constant force field does zero work on a particle that moves once uniformly around the circle* $x^2 + y^2 = 1$.

$\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$, $\quad \vec{F} = \langle a, b \rangle \quad a, b$ constants

$0 \le t \le 2\pi$

$$W = \int_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \langle a, b\rangle \cdot \langle -\sin(t), \cos(t)\rangle\, dt = \int_0^{2\pi} -a\sin(t) + b\cos(t)\, dt$$

$$= a\cos(t) + b\sin(t)\Big|_0^{2\pi} = \left(a\cos(2\pi) + \cancel{b\sin(2\pi)}^{\,0}\right) - \left(a\cos(0) + \cancel{b\sin(0)}^{\,0}\right)$$

$$= a\,(\underbrace{\cos(2\pi) - \cos(0)}_{=0})$$

$$= 0$$