# Math 53 - Multivariable Calculus

Quiz # 8

October 27th, 2011

solns

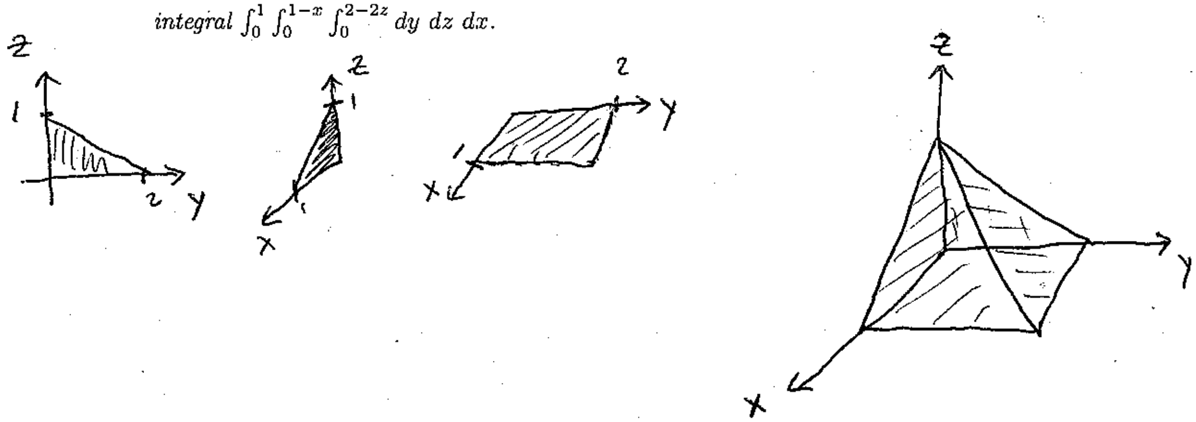
**Exercise 1.** *Use cylindrical coordinates to find the volume of the region $E$ bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.*

Paraboloids intersect when $x^2+y^2 = z = 36-3x^2-3y^2 \Rightarrow x^2+y^2=9$

$\Rightarrow$ integrating over $\{(x,y) \mid x^2+y^2 \le 9\}$. This becomes, in cylindrical coords.

$$V = \int_0^{2\pi}\int_0^3\int_{r^2}^{36-3r^2} r\,dz\,dr\,d\theta = 162\pi.$$

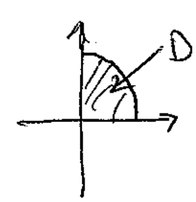
**Exercise 2.** *Describe (with a few words AND a sketch) the solid whose volume is given by the iterated integral $\int_0^1\int_0^{1-x}\int_0^{2-2z} dy\;dz\;dx$.*




**Exercise 3.** *Evaluate $\int\int_R \sin(9x^2+4y^2)dA$, where $R$ is the region in the first quadrant bounded by the ellipse $9x^2+4y^2=1$.*

Let $u = 3x,\ v = 2y \Rightarrow 9x^2+4y^2 = u^2+v^2$ and $x = \frac{1}{3}u,\ y = \frac{1}{2}v$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix} = \begin{vmatrix} 1/3 & 0 \\ 0 & 1/2\end{vmatrix} = 1/6.$$ Further, $R$ becomes




$$\Rightarrow \iint_R \sin(9x^2+4y^2)\,dA = \iint_D \sin(u^2+v^2)\left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv = \int_0^{\pi/2}\int_0^1 \frac{1}{6}\sin(r^2)\,r\,dr\,d\theta$$

$$= \frac{\pi}{24}\left(1-\cos(1)\right)$$