Math 53 - Multivariable Calculus

Quiz # 7

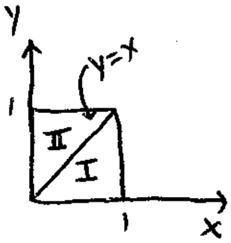
October 19th, 2011

Solns

Exercise 1. *Evalute the integral*

$$\int_0^1 \int_0^1 e^{max\{x^2,y^2\}} dydx,$$

where $max\{x^2, y^2\}$ *means the larger of the numbers* x^2 *and* y^2. *(Hint, disect the region of integration along the line* $y = x$.*)*



In region I, $x>y \Rightarrow x^2>y^2 \Rightarrow \max\{x^2,y^2\} = x^2$. So, we're integrating $\iint_I e^{x^2} dA = \int_0^1\int_0^x e^{x^2} dy\,dx = \int_0^1 x e^{x^2} dx = \frac{1}{2}e - \frac{1}{2}$. For region II, $y>x \Rightarrow y^2>x^2 \Rightarrow \max\{x^2,y^2\} = y^2$. Therefore, we're integrating $\iint_{II} e^{y^2} dA = \int_0^1\int_0^y e^{y^2} dx\,dy = \frac{1}{2}e - 1/2$. So, we have

$$\int_0^1\int_0^1 e^{\max\{x^2,y^2\}} dydx = \left(\tfrac{1}{2}e - \tfrac{1}{2}\right) + \left(\tfrac{1}{2}e - \tfrac{1}{2}\right) = e-1$$

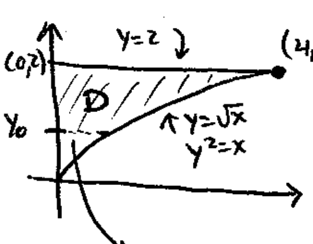
Exercise 2. *Evaluate* $\int\int_R e^{-x^2-y^2} dxdy$, *where* $R = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1\}$.

$$\iint_R e^{-x^2-y^2} dxdy = \iint_R e^{-(x^2+y^2)} dxdy = \int_0^{2\pi}\int_0^1 e^{-r^2} r\,dr\,d\theta$$

$$\left(\text{Let } u=-r^2 \Rightarrow du = -2r\,dr \Rightarrow \int e^u du = e^u = e^{-r^2}\Big|_0^1 = e^{-1}-1\right)$$

$$= -\frac{1}{2}\int_0^{2\pi} (e^{-1}-1)\,d\theta = \pi(e^{-1}-1).$$

Exercise 3. *Evaluate* $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1} dydx$. *(Hint, reverse the order of integration.)*



for fixed Y_0,
$0 \leq X_0 \leq Y_0^2$

$$\int_0^4\int_{\sqrt{x}}^2 \frac{1}{y^3+1} dydx = \int_0^2\int_0^{y^2} \frac{1}{y^3+1} dx\,dy$$

$$= \int_0^2 \frac{y^2}{y^3+1} dy = \frac{1}{3} \ln|y^3+1| \Big|_0^2 = \frac{1}{3}\ln(9)$$

$u = y^3+1$