Math 53 - Multivariable Calculus

Quiz # 6

October 6th, 2011

Solns

Exercise 1. *Find the equation of the plane containing the three points $P_0 = (2,1,0)$, $P_1 = (1,0,1)$, $P_2 = (2,-1,1)$. Also, find the intersection of this plane with the line parallel to the vector $\vec{V} = \langle 1,1,1\rangle$ and passing through the point $S = (-1,0,0)$.*

Eq$\underline{n}$ of plane is $\overrightarrow{P_0P} \cdot \vec{N} = 0$. So, let $P = (x,y,z) \Rightarrow \overrightarrow{P_0P} = \langle x-2, y-1, z-0\rangle$

$\vec{N} = \overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2} = \langle 1,1,2\rangle \Rightarrow \overrightarrow{P_0P} \cdot \vec{N} = 0 \Longleftrightarrow \boxed{x+y+2z=3.}$

Next, line is given by $x = -1+t$, $y = t$, $z = t$. Subing these into the eq$\underline{n}$ of the plane gives $-1+t+t+2t = 3 \Longleftrightarrow t = 1$. Hence, the intersection point is $(-1+1, 1, 1) = \boxed{(0,1,1).}$

Exercise 2. *Find the equation of the tangent plane to the surface $x^3y + z^2 = 3$ at the point $(-1,1,2)$.*

Can get $\vec{N}$ for plane by $\vec{N} = \nabla f\big|_{(-1,1,2)} = \langle 3,-1,4\rangle$.

So, LHS of eq$\underline{n}$ is $3x - y + 4z$. Plugging in the point $(-1,1,2)$ gives $3(-1) - (1) + 4(2) = 4 \Rightarrow$ eq$\underline{n}$ of plane is

$$3x - y + 4z = 4.$$

Exercise 3. *Suppose $(1,1)$ is a critical point of a function f with continuous second derivatives. What can you say about f given that $f_{xx}(1,1) = 4$, $f_{xy}(1,1) = 1$, and $f_{yy}(1,1) = 2$.*

Since $D = f_{xx}(1,1)f_{yy}(1,1) - f_{xy}(1,1)^2 = 8 - 1 = 7 > 0$ and since $f_{xx}(1,1) = 4 > 0$, f has a local minimum at $(1,1)$.