Math 53 - Multivariable Calculus Solns

Quiz # 5

September 29th, 2011

Exercise 1. *Find the linear approximation of the function* $f(x,y) = \sqrt{20 - x^2 - 7y^2}$ *at* $(2,1)$ *and use it to approximate* $f(1.95, 1.08)$.

$$f_x(x,y) = -\frac{x}{\sqrt{20-x^2-7y^2}}, \quad f_y(x,y) = \frac{-7y}{\sqrt{20-x^2-7y^2}}$$

$$\Rightarrow f_x(2,1) = -2/3, \quad f_y(2,1) = -7/3 \Rightarrow f(2,1) \approx -\frac{2}{3}x - \frac{7}{3}y + \frac{20}{3}$$

$$\Rightarrow f(1.95, 1.08) \approx -\frac{2}{3}(1.95) - \frac{7}{3}(1.08) + \frac{20}{3}$$

Exercise 2. *Use linear approximation to estimate the amount of tin is a closed tin can with diameter* $8cm$ *and height* $12cm$ *if the tin is* $0.04cm$ *thick.*

$$V = \pi r^2 h, \quad \Delta V \approx 2\pi r \Delta r + \pi r^2 \Delta h.$$

$$\Delta r = 0.04, \quad \Delta h = \underbrace{0.04}_{\text{top}} + \underbrace{0.04}_{\text{bottom}} = 0.08, \quad r = 4cm, \quad h = 12cm$$

$$\Delta V \approx 2\pi(4)(12)(0.04)\,cm^3 + \pi(16)(0.08)\,cm^3$$

$$\Rightarrow \Delta V \approx 16.08\,cm^3$$

Exercise 3. *If* $z = f(x-y)$, *show that* $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$. *(Hint, let* $u = x - y$ *and then use the chain rule to compute* $\frac{\partial z(u)}{\partial x}$ *and* $\frac{\partial z(u)}{\partial y}$.)

Let $u = x - y$, then $\frac{\partial z}{\partial x} = \frac{dz}{du}\underbrace{\frac{\partial u}{\partial x}}_{=1} = \frac{dz}{du}$ and

$$\frac{\partial z}{\partial y} = \frac{dz}{du}\underbrace{\frac{\partial u}{\partial y}}_{=-1} = -\frac{dz}{du} \Rightarrow \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{dz}{du} - \frac{dz}{du} = 0.$$