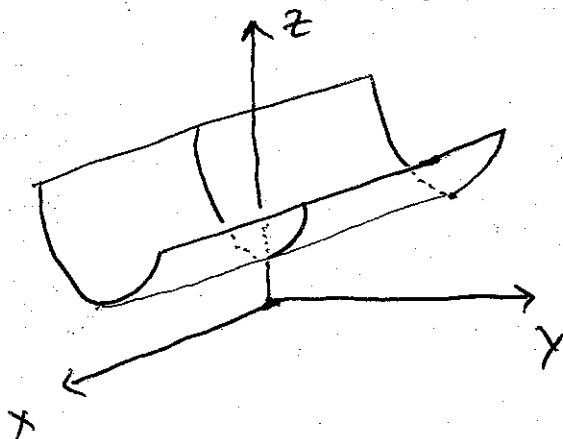# Math 53 - Multivariable Calculus

$\underline{Solns}$

Exercise 1. *Find the velocity, acceleration, and speed of a particle whose position vector is $\vec{r}(t) = \langle \sqrt{2}t, e^t, e^{-t} \rangle$.*

$$\vec{v}(t) = \vec{r}'(t) = \langle \sqrt{2}, e^t, -e^t \rangle \quad \Rightarrow \quad speed = |\vec{v}(t)| = \sqrt{e^{2t} + e^{-2t} + 2}$$

$$\vec{a}(t) = \vec{v}'(t) = \langle 0, e^t, e^{-t} \rangle \qquad = \sqrt{(e^t + e^{-t})^2}$$

$$= e^t + e^{-t}$$

Exercise 2. *Sketch the graph of the function $f(x,y) = y^2 + 1$.*

Exercise 3. *Is $u(x,t) = \sin(x - at) + \ln(x + at)$ a solution to $u_{tt} - a^2 u_{xx} = 0$?*

$$U_t = -a\cos(x-at) + \frac{a}{x+at} \quad \Rightarrow \quad U_{tt} = -a^2\sin(x-at) - \frac{a^2}{(x+at)^2}$$

$$U_{xx} = -\sin(x-at) - \frac{1}{(x-at)^2}$$

$$\Rightarrow \quad U_{tt} - a^2 U_{xx} = 0 \quad \Rightarrow \quad U(x,t) \ \text{is a soln.}$$