

Math 53 - Multivariable Calculus

Quiz # 3

Solns

September 15th, 2011

Exercise 1. Find the volume of the parallelepiped determined by the vectors $\vec{A} = \langle 6, 3, -1 \rangle$, $\vec{B} = \langle 0, 1, 2 \rangle$, and $\vec{C} = \langle 4, -2, 5 \rangle$.

$$\text{Vol} = \vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} 6 & 3 & -1 \\ 0 & 1 & 2 \\ 4 & -2 & 5 \end{vmatrix} = 6(5+4) - 3(0-8) - (0-4) = 82$$

$$\Rightarrow \text{Vol} = 82 \text{ (units)}^3$$

Exercise 2. Find the equation of the plane that passes through the point $(6, 0, -2)$ and contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$.

Need to find two nonparallel vectors. Since the line is in plane $\Rightarrow \vec{V}_1 = \langle -2, 5, 4 \rangle$ in plane. To find \vec{V}_2 pick any point on line and then \vec{V}_2 will be vector connecting ~~the~~ ^{this} point w/ $(6, 0, -2)$. Setting $t=0$ gives $(4, 3, 7)$

$$\Rightarrow \vec{V}_2 = \langle 6-4, 0-3, -2-7 \rangle = \langle 2, -3, -9 \rangle. \text{ Now } \vec{N} \text{ for our plane is}$$

$$\vec{N} = \vec{V}_1 \times \vec{V}_2 = \langle -33, -10, -4 \rangle \Rightarrow \text{LHS of the eqn for our plane is}$$

$$-33x - 10y - 4z \text{ and RHS is } -33(6) - 10(0) - 4(-2) = -190$$

$$\Rightarrow \boxed{-33x - 10y - 4z = -190}$$

Exercise 3. Determine whether the planes $x + 4y - 3z = 1$ and $-3x + 6y + 7z = 0$ are perpendicular, parallel, or neither. If neither, find the angle between them.

$$\vec{N}_1 = \langle 1, 4, -3 \rangle \quad \vec{N}_2 = \langle -3, 6, 7 \rangle$$

$$\vec{N}_1 \cdot \vec{N}_2 = -3 + 24 - 21 = 0$$

\Rightarrow planes are \perp