Math 53 - Multivariable Calculus

Quiz # 1

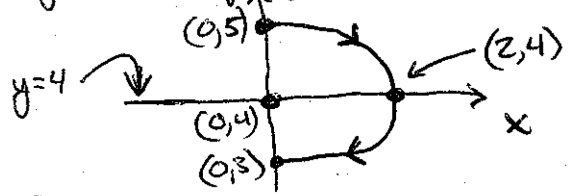
Solns

September 1st, 2011

Exercise 1. *Describe the motion of a particle with position $(x(t), y(t))$, where $x(t) = 2\sin(t)$, $y(t) = 4 + \cos(t)$ and $0 \le t \le \pi$.*

$x(t) = 2\sin(t) \Rightarrow \sin(t) = \frac{x(t)}{2}$, $y(t) = 4 + \cos(t) \Rightarrow \cos(t) = y(t) - 4$

Since $\cos^2(t) + \sin^2(t) = 1 \Rightarrow \left(\frac{x(t)}{2}\right)^2 + \left(\frac{y(t)-4}{1}\right)^2 = 1$. So, since $0 \le t \le \pi$, we get the right half of an ellipse centered at $(0,4)$:



Exercise 2. *Set up an integral, but do not evaluate, that represents the length of the curve given by $x(t) = t - t^2$, $y(t) = \frac{4}{3}t^{3/2}$ and $1 \le t \le 2$.*

The length is $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$, $\frac{dx}{dt} = 1 - 2t$, $\frac{dy}{dt} = 2t^{1/2}$ and $1 \le t \le 2$. So, we have

$$L = \int_1^2 \sqrt{(1-2t)^2 + (2t^{1/2})^2}\, dt = \int_1^2 \sqrt{1+4t^2}\, dt$$

Exercise 3. *Identify the following curve,*

$$r = 2\sin(\theta) + 2\cos(\theta), \qquad 0 \le \theta < 2\pi,$$

by finding a Cartesian equation of the curve. (Hint: Start by multiplying both sides by r, then convert to Cartesian coordinates and complete the square.)

$r = 2\sin(\theta) + 2\cos(\theta) \Rightarrow r^2 = 2r\sin(\theta) + 2r\cos(\theta) \Rightarrow x^2 + y^2 = 2x + 2y$

$\Rightarrow x^2 - 2x + y^2 - 2y = 0 \Rightarrow (x^2 - 2x + 1) + (y^2 - 2y + 1) - 2 = 0$

$\Rightarrow (x-1)^2 + (y-1)^2 = 2 \Rightarrow$ the curve is a circle of radius $\sqrt{2}$ centered at $(1,1)$.