Math 53 - Multivariable Calculus

Quiz # 12

December 1st, 2011

Exercise 1. *Let c be a simple, closed smooth curve and let Σ_1 and Σ_2 be two smooth surfaces (with positive orientation) with $\partial\Sigma_1 = \partial\Sigma_2 = c$. Additionally, let $\vec{\boldsymbol{F}}$ be a vector field on $\mathbb{R}^3$ whose components have continuous partial derivatives. Explain why or why not $\int\int_{\Sigma_1} curl(\vec{\boldsymbol{F}}) \cdot d\mathbf{S} = \int\int_{\Sigma_2} curl(\vec{\boldsymbol{F}}) \cdot d\mathbf{S}$.*

Exercise 2. *Suppose a particle moves along line segments from the origin to the points $(1, 0, 0)$, $(1, 2, 1)$, $(0, 2, 1)$, and back to the origin under the influence of the force field $\vec{\boldsymbol{F}} = \langle z^2, 2xy, 4y^2 \rangle$. Find the work done. (HINT: Use Stokes' theorem)*

Exercise 3. *Let S_a^2 denote the sphere of radius a and suppose the vector field $\vec{\boldsymbol{F}}$ satisfies the hypotheses of Stokes' theorem. Show that $\int\int_{S_a^2} curl(\vec{\boldsymbol{F}}) \cdot d\vec{\boldsymbol{S}} = 0$.(HINT: Decompose S_a^2 as the union of the northern hemisphere and the southern hemisphere. Then apply Stokes' theorem on each piece (cough careful about orientations cough)).*