Math 53 - Multivariable Calculus

Quiz # 11

November 17th, 2011

Solns

Exercise 1. *Find a parametric represenation for the surface given by the plane that passes through the point* $(1, 2-3)$ *and contains the vectors* $\hat{\imath}+\hat{\jmath}-\hat{k}$ *and* $\hat{\imath}-\hat{\jmath}+\hat{k}$.

We simply follow example 3 on p 1072: $x = 1 + u(1) + v(1)$,

$y = 2 + u(1) + v(-1)$, $z = -3 + u(-1) + v(1)$

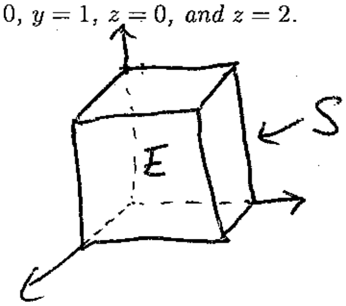
$$\Rightarrow \text{parametric eqns are } \begin{cases} x = 1 + u + v \\ y = 2 + u - v \\ z = -3 - u + v \end{cases}$$

Exercise 2. *Evaluate the surface integral* $\int\int_S x^2yz\ dS$, *where* S *is the part of the plane* $z = 1 + 2x + 3y$ *that lies above the rectangle* $[0, 3] \times [0, 2]$.

$$\iint_S x^2yz\,dS = \iint_D x^2yz\sqrt{(z_x)^2+(z_y)^2+1}\,dA = \int_0^3\int_0^2 x^2y(1+2x+3y)\sqrt{14}\,dy\,dx$$

$$= \int_0^3 \sqrt{14}(10x^2+4x^3)\,dx = 171\sqrt{14}$$

Exercise 3. *Use the divergence theorem to calculate the flux of* $\vec{F} = \langle e^x\sin(y), e^x\cos(y), yz^2\rangle$ *across* S, *where* S *is the surface of the box bounded by the planes* $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, *and* $z = 2$.



$$\operatorname{div}(\vec{F}) = e^x\sin(y) - e^x\sin(y) + 2yz = 2yz$$

$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{div}(\vec{F})\,dV = \int_0^1\int_0^1\int_0^2 2yz\,dz\,dy\,dx$$

$$= 2\int_0^1 dx \int_0^1 y\,dy \int_0^2 dz = 2$$