Math 53 - Multivariable Calculus

Practice Midterm # 2

November 17th, 2011

Exercise 1. *Evaluate* $\int\int_R e^{x+y}\, dA$, *where* R *is given by the inequality* $|x| + |y| \leq 1$.

Exercise 2. *Let* $\vec{F} = \nabla f$, *where* $f(x,y) = \sin(x - 2y)$. *Find a curve* c *such that* $\int_c \vec{F} \cdot d\vec{r} = 1$.

Exercise 3. *Suppose you have a vector field* $\vec{F} = \langle M(x,y), N(x,y) \rangle$ *which has continuous first partial derivatives on* $\mathbb{R}^2 \setminus \{(a,b)\}$ *such that* $M_y = N_x$ *on all of* $\mathbb{R}^2 \setminus \{(a,b)\}$. *Prove that* $\vec{F}$ *is conservative, or give a counterexample.*

Exercise 4. *Let* $S = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$ *denote the unit sphere in* $\mathbb{R}^3$. *Evaluate the following surface integral over* S:

$$\int\int_S (x^2 + y + z)\, dA.$$

Exercise 5. *Let M be the solid cylinder $x^2 + y^2 \leq 1$, $0 \leq z \leq 1$ and let $\vec{\mathbf{F}} = \langle xy, yz, zx \rangle$. Verify that the divergence theorem is true for $\vec{\mathbf{F}}$ on the region M.*

Exercise 6. (*Very Hard!*) *Let*

$$\vec{F} = \frac{(2x^3 + 2xy^2 - 2y)\hat{\imath} + (2y^3 + 2x^2y + 2x)\hat{\jmath}}{x^2 + y^2}.$$

Evaluate $\int_c \vec{F} \cdot d\vec{r}$ *where c is shown below:*