

- send John an email letting him know if he will be talking next week

- Let $G = SU(2)$

P
 \downarrow w/ connection A , $F = dA + A \wedge A$
 M^3

can expand $\det\left(I + \frac{iF}{2\pi}\right) = C_0 + C_1 + C_2 + \dots$

where $C_2 = \frac{1}{8\pi^2} \text{Tr}(F \wedge F)$

want 3-form C_3 such that $dC_3 = C_2$

We propose $C_3 = A \wedge dA + \frac{2}{3} A \wedge A \wedge A$. Then can check

$$dC_3 = \text{Tr} \left[\frac{1}{2} F^2 - A^2 F - FA^2 + A^4 + \frac{2}{3} (FA^2 - AFA + A^2 F - A^4) \right]$$

$$= \text{Tr}(F^2)$$

Need to know how G acts on the CS form C_3 .
 Under $\varphi: P \rightarrow P$, get $\theta \in \mathfrak{g}: P \rightarrow \mathfrak{g}$ and can pull down to M using

~~$\text{let } \varphi^*(A + \text{Ad}_{g^{-1}}(A) + \varphi^*(\theta))$~~

section. We'll see that $A \mapsto \text{Ad}_{g^{-1}}(A) + \varphi^*(\theta)$

\hookrightarrow M.C. form

This tells us:

$$\varphi^* C_3 = C_3 + d(\text{Ad}_{\varphi^{-1}}(A) \wedge \varphi^*(\theta)) - \frac{1}{3} \text{Tr}(\varphi^* \theta \wedge \varphi^* \theta \wedge \varphi^* \theta)$$

for closed $M \Rightarrow \int_M \varphi^* C_3 - \int_M C_3 = \underbrace{\frac{1}{3} \int_M \text{Tr}(\varphi^* \theta^3)}_{\in \mathbb{Z}}$

$\Rightarrow e^{2\pi i \int C_3}$ is well-defnd

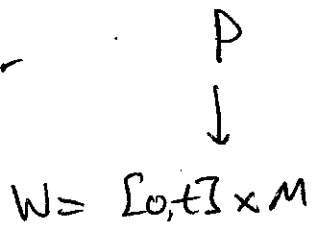
Defⁿ the CS action as $\int_M p^* C_3(A) = S(p, A)$ section

Now, if $M = M_1 \cup M_2$ w/ A_1 and A_2 then

$$S(p, A) = S(p_1, A_1) + S(p_2, A_2)$$

Can drop p dependence since any other section is related via a gauge transformation.

consider



A_t , where A_t restricts to A on boundary

$$S(A_t) - S(A_0) = S_{\text{DW}}(A_t) = \int_0^t \int_M (F_t + ds \wedge \dot{A}_s)^2$$

$$= 2 \int_0^t \int_M F_s \wedge \dot{A}_s$$

Want extrema: So need $F_s \equiv 0$

So, we want moduli space of flat connections

We can show, this space is

$$\text{Hom}(\pi_1(M), G) / G$$

or, reducing to ~~the~~ surfaces Y^2



Flat connection on Y^2 gives flat connections on $Y^2 \times [0, \infty)$

Let $A_Q =$ space all connections on Q , then $T_A A_Q \cong \mathcal{J}^1(Y, \mathfrak{g})$

Defⁿ
 $\omega(a, b) = \int_Y \text{Tr} a \wedge b$, gives symplectic form on A_Q

The space A_Q is too large - need to perform symplectic reduction.

$$\text{defn } \mu: A_Q \rightarrow (\text{Lie } G_Q)^*$$

$$\parallel$$

$$\int^2(Y, \omega)$$

We already know of a way to go from $A_Q \rightarrow \int^2(Y, \omega)$, the covariate.

$$\langle d\mu|_{A(Q)} \epsilon \rangle = \int_{X_\epsilon} \omega(a)$$

$$\parallel \qquad \parallel$$

$$\int_Y \epsilon d_A a$$

$$\int_Y d_A \epsilon a$$

~~the~~

the space we want is $\mu^{-1}(0)/G$

Let's see how much smaller this got.

$$0 \rightarrow \int^0(Y, \omega) \xrightarrow{d_A} \int^1(Y, \omega) \xrightarrow{d_A} \int^2(Y, \omega)$$

~~A_Q~~

$$T_A \mathcal{M}_Q \cong H^1(\mathcal{R}^1(Y, \mathcal{O}_Y))$$

↓
moduli space of flat connections

$$\underbrace{\dim H^0}_{\text{equal}} - \dim H^1 + \underbrace{\dim H^2}_{\text{equal}} = \dim G \cdot \chi(Y)$$

$$\Rightarrow \dim H^1 = -\dim G \cdot \chi(Y) + 2 \dim H^0$$

Again consider

Let I be a complex structure on Y , then can divide:

$$\mathcal{R}^1(Y, \mathcal{O}_Y) = \mathcal{R}^{0,1} \oplus \mathcal{R}^{1,0}$$

$$d_A = \partial_A + \bar{\partial}_A$$

Let A_0 be fixed, then

$$\bar{\partial}_A = \bar{\partial}_0 + \text{something in } \mathcal{R}^{0,1}$$

so can ~~divide~~ quotient by $\text{Aut}(\mathcal{O}_Y \otimes \mathbb{C})$ giving another

moduli space M_I , and can prove $M_I \cong M_Q$