MATH 32 FALL 2012
MIDTERM 2 - PRACTICE EXAM SOLUTIONS

(1) (3 points each) Evaluate the following expressions:

(a) $\log_{16}(32)$

Changing base to base 2, $\log_{16}(32) = \dfrac{\log_2(32)}{\log_2(15)} = \dfrac{5}{4}$.

(b) $8^{-\frac{5}{3}}$

$$8^{-\frac{5}{3}} = \frac{1}{8^{\frac{5}{3}}} = \frac{1}{\sqrt[3]{8}^5} = \frac{1}{2^5} = \frac{1}{32}$$

(c) $e^{2\ln(3)+\ln(5)}$

$$e^{2\ln(3)+\ln(5)} = e^{\ln(3^2)+\ln(5)} = e^{\ln(9\cdot5)} = 45$$

(d) area $\left(\frac{1}{x}, 1, e^2\right)$

$$\text{area}\left(\frac{1}{x}, 1, e^2\right) = \ln(e^2) = 2$$

(2) (a) Write an equation for a circle with center $(1, 3)$ and radius 4.

$$(x-1)^2 + (y-3)^2 = 16$$

(b) What is its circumference?

$$2\pi r = 2\pi \cdot 4 = 8\pi$$

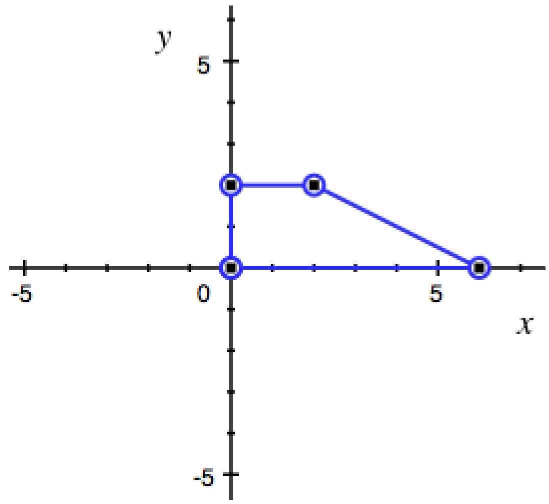
(c) What is its area?

$$\pi r^2 = \pi \cdot 4^2 = 16\pi$$

(3) Consider a trapezoid with vertices $(0, 0)$, $(6, 0)$, $(0, 2)$, and $(2, 2)$.

(a) What is its perimeter?

Let's draw a picture:



The lengths of three of the sides are easy to compute: they are 6 (from $(0,0)$ to $(6,0)$), 2 (from $(0,0)$ to $(0,2)$), and 2 (from $(0,2)$ to $(2,2)$).
For the last side (from $(6,0)$ to $(2,2)$), we need the distance formula:

$$\sqrt{(2-6)^2+(2-0)^2} = \sqrt{(-4)^2+2^2} = \sqrt{20}$$

So the perimeter is $6+2+2+\sqrt{20} = 10+\sqrt{20}$.

(b) What is its area?

The area of a trapezoid is $A = \dfrac{(b_1+b_2)}{2} \cdot h$. Here, the lengths of the bases are 6 and 2, and the height is 2 (for most trapezoids, the height will not be the length of one of the sides - for this one, conveniently, it is the length of the left-most side).
So the area is $\dfrac{6+2}{2} \cdot 2 = 8$.

(4) You deposit \$50 in a bank account which promises an annual interest rate of 5%.

(a) Write an expression for the amount of money in the account after t years have passed, assuming that the bank compounds interest monthly.

$$A(t) = 50\left(1+\frac{.05}{12}\right)^{12t}$$

(b) Write an expression for the amount of money in the account after t years have passed, assuming that the bank compounds interest continuously.

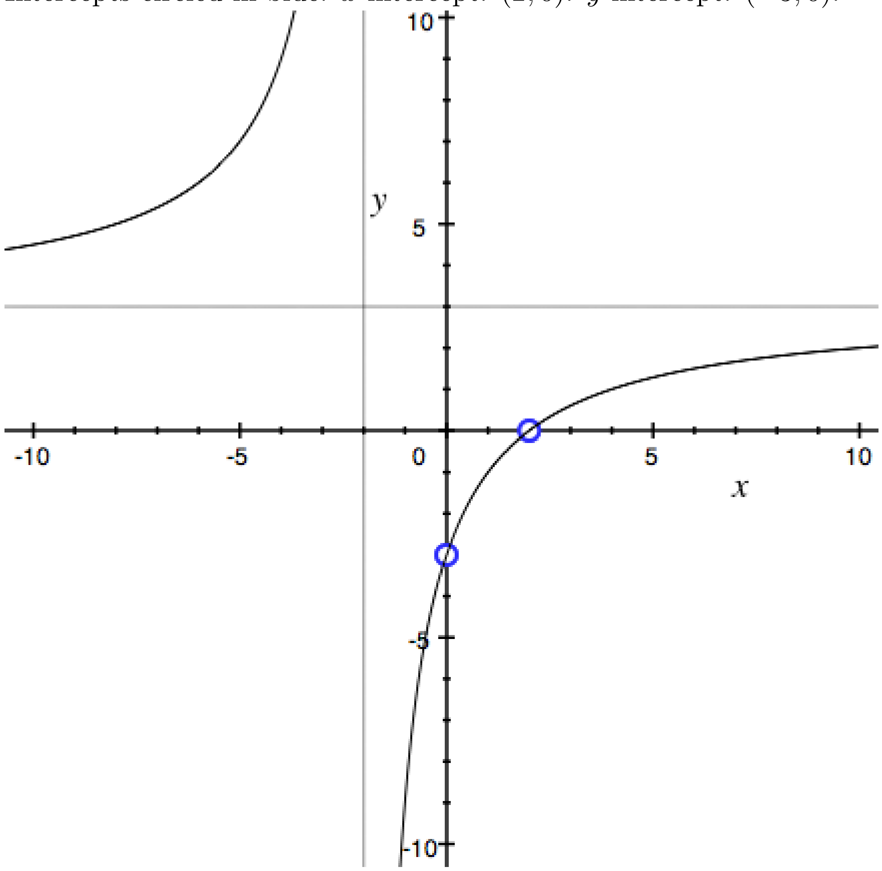
$$A(t) = 50e^{.05t}$$

(c) If interest is compounded continuously, how long will it take before you have \$200?

Setting $A(t) = 200$ in the above equation, we get $200 = 50e^{.05t}$.
Solving for t, $e^{.05t} = \frac{200}{50} = 4$, $.05t = \ln(4)$, $t = \frac{\ln(4)}{.05} = 20\ln(4)$ years.

(5) Sketch a graph of the function $f(x) = \dfrac{3x-6}{x+2}$, labeling all horizontal and vertical asymptotes and x and y intercepts.

We can write this as $\dfrac{3(x-2)}{x+2}$. The zero of the numerator (x-intercept) is 2. The zero of the denominator (vertical asymptote) is -2. The ratio of leading terms in numerator and denominator (horizontal asymptote) is 3. To find the y-intercept, evaluate $f(0) = \frac{-6}{2} = -3$.
Graph plotted in black.
Asymptotes plotted in grey. Vertical: $x = -2$. Horizontal: $y = 3$.
Intercepts circled in blue. x-intercept: $(2, 0)$. y-intercept: $(-3, 0)$.



(6) Which number is greater, $\log_2(5)$ or $\log_3(8)$? Explain.

Although it's hard to approximate these numbers without a computer, we can notice the following: $2^2 = 4$, so $\log_2(4) = 2$, and $2 < \log_2(5)$. On the other hand, $3^2 = 9$, so $\log_3(9) = 2$, and $\log_2(8) < 2$.
Hence $\log_2(5)$ is greater than $\log_3(8)$.