MATH 32 MIDTERM 1 REVIEW

Office Hours this week

Mike (1061 Evans): Tuesday 1-2, Thursday 1-3.
Alex (787 Evans): Wednesday 12-1 and 3-4, Thursday 11-12.

Material

The exam covers material from lecture up to and including Section 3.5 of the textbook. Specifically: 1.1-4, 1.6-7, 2.1-4, 3.1-5. Section 3.6 will not be included.

Exponents and Roots

- You should be comfortable with the various properties of exponents and roots, see pp. A-9, A-10, A-15, A-22.
- $\sqrt[3]{x}$ is defined for any x, and the same holds for all odd roots. But $\sqrt[4]{x}$ is only defined for $x \geq 0$, and the same holds for all even roots.
- $x^2 = 4$ has two solutions, $x = \pm 2$. But the notation $\sqrt{4}$ refers *only* to the positive root, 2. $x^3 = 8$ has only one solution.
- For this reason, $\sqrt[2]{x^2} = |x|$, but $\sqrt[3]{x^3} = x$.

Simplifying Expressions

- You should be able to factor reducible quadratic polynomials and some nice higher degree (i.e. cubic and quartic) polynomials. See p. A-26 for factoring strategies.
- You may need to work with fractional expressions. This includes putting fractions over a common denominator, adding, multiplying, and reducing to lowest terms. See Section B.5 for examples.
- If an expression has an absolute value, think about whether the term inside the absolute value is positive or negative. If you know the answer, you can remove the absolute value signs (adding a $-$ if the term is negative). If you don't know the answer, you may have to break the problem into cases and/or multiple equations/inequalities.

Lines and Distances

- You should be able to solve linear equations, i.e. $ax + b = cx + d$.
- Given two points, (a_1, b_1) and (a_2, b_2), the equation for the line through them is $(y - b_1) = m(x - a_1)$, where the slope m is $\dfrac{b_2 - b_1}{a_2 - a_1}$. If $a_1 = a_2$, the slope is not defined, and the line is the vertical line $x = a_1$.
- The distance from (a_1, b_1) to (a_2, b_2) is $\sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}$. Remember that this comes from the Pythagorean theorem $a^2 + b^2 = c^2$ for right triangles.

Circles and Quadratics

- The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. You should be able to put an equation for a circles into standard form so that you can read off its center and radius by completing the square.

- To complete the square, given x^2+ax, add $\left(\frac{a}{2}\right)^2$ to get $x^2+ax+\left(\frac{a}{2}\right)^2 = \left(x+\frac{a}{2}\right)^2$. Make sure you also subtract the constant you added, or add it to the other side of an equation!
- To solve quadratics $ax^2+bx+c=0$, factor as $(x-r_1)(x-r_2)=0$, in which case the solutions are r_1 and r_2, or use the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$.
- The term inside the square root in the quadratic formula is called the discriminant. It provides a test for the number of roots: If b^2-4ac. If $b^2-4ac>0$, the quadratic has two roots. If $b^2-4ac=0$, the quadratic has one root. If $b^2-4ac<0$, the quadratic has no roots.
- Sometimes more complicated equations are really quadratics in disguise! See Section 2.2 for examples of simplifications and substitutions that can help you turn harder equations into quadratics.

Intervals and Inequalities

- You should be comfortable with interval notation and how to translate between intervals given as inequalities, given in interval notation, and drawn on a number line. For examples, see pp. 3-4.
- When solving inequalities, remember to
  - Keep track of which direction the inequality is facing.
  - Remember the difference between $<$ and $\leq$.
  - Remember to switch the direction of the inequality when you multiply both dies by a negative number.
- When an inequality involves absolute values, you may have to split into cases. Think carefully about how you put together the solutions at the end of the problem!
- If an inequality involves a polynomial, find the roots of the polynomial. To find out when the polynomial is positive or negative, you just need to check a value in each region between the roots.

Functions and Graphing

- A function assigns a *unique* element of its range to each element in its domain.
- The domain of a function on the real numbers is (for us) the maximum set of real numbers on which the description of the function is defined. The range of a function is exactly the set of values the function takes on.
- You can compute the average rate of change of a function f on the interval $[a,b]$ by $\dfrac{f(b)-f(a)}{(b-a)}$.
- Many functions we encounter will be obtained from simpler functions in a few simple ways. See p.172 for a description of how changes to formulas correspond to changes in graphs.
- There are other ways of combining functions, by arithmetic or composition. See pp. 182-183 for examples.