

Qualifying Exam Syllabus

Alex Kruckman

June 12, 2012

Committee: George Bergman, Martin Olsson (chair), Sherrilyn Roush (philosophy), Thomas Scanlon.

Major Topic: Model Theory (Foundations)

Basic constructions: The completeness and compactness theorems. The Löwenheim-Skolem theorem. Omitting types. Ultraproducts. Indiscernible sequences.

Nice properties: Saturation, homogeneity, and the monster model. Elimination of imaginaries and M^{eq} . Quantifier elimination. Model completeness. Countably categorical theories.

Stable theories: Characterizations of a stable formula. Stable, superstable, and ω -stable theories. Forking via definability of types. Forking via dividing (in general theories). Properties of the nonforking independence relation. Canonical bases.

Ranks: φ -2-rank, Morley rank, R^∞ -rank, U-rank.

Pregeometries: Dimension. Trivial, modular, and locally modular pregeometries. The geometry associated to a pregeometry. The pregeometry on a minimal set. Plane curves and linearity.

Major Topic: Algebraic Geometry (Algebra)

Sheaves: Presheaves and sheaves. Sheafification. Pushforward and inverse image sheaves.

Schemes: The Spec and Proj constructions. Gluing of schemes. Conditions for representability of functors. Fiber product. Irreducible, reduced, integral, and (locally) noetherian schemes.

Morphisms of schemes: Open and closed immersions. Finite, finite type, quasiseparated, separated, proper, and affine morphisms. Scheme theoretic image. The functor of points.

Sheaves of Modules: Quasicoherent, coherent, and locally free sheaves. The Picard group. Ample and very ample line bundles.

Cohomology: Derived functor and Čech cohomology of sheaves. Cohomology of projective space. Grothendieck's Vanishing Theorem. Serre duality (statement).

Curves: The Riemann-Roch theorem for curves. Degree, ramification, and the Riemann-Hurwitz formula. Embeddings into projective space.

Minor Topic: Category Theory (Algebra)

Basics: Categories, functors, natural transformations. Limits and colimits. Epics and monics. Yoneda's lemma and representable functors. Adjunctions. Freyd's adjoint functor theorem.

Universal algebra: Varieties of algebras and functors between them. Algebra objects in categories. Clonal categories / Lawvere theories.

Homological Algebra: Abelian categories. Complexes, homology, and homotopy. Injective and projective objects. Derived functors.

Foundational issues: Large, small, and locally small categories. The Axiom of Universes and its relationship to ZFC.