MATH 32 FALL 2012
MIDTERM 1 - PRACTICE EXAM SOLUTIONS

(1) Find all values of x satisfying the inequality

$$\left|\frac{1}{x}\right| \geq 5$$

Solution: Case 1: $\frac{1}{x} \geq 0$. This happens when $x > 0$. Then $\left|\frac{1}{x}\right| = \frac{1}{x}$.

$$\begin{aligned} \frac{1}{x} \geq 5 \quad &\Longleftrightarrow \quad 1 \geq 5x \\ &\Longleftrightarrow \quad x \leq \frac{1}{5} \end{aligned}$$

Case 2: $\frac{1}{x} < 0$. This happens when $x < 0$. Then $\left|\frac{1}{x}\right| = -\frac{1}{x}$.

$$\begin{aligned} -\frac{1}{x} \geq 5 \quad &\Longleftrightarrow \quad -1 \leq 5x \\ &\Longleftrightarrow \quad -\frac{1}{5} \leq x \end{aligned}$$

Putting these cases together, we see that the solutions are $\left[-\frac{1}{5}, 0\right) \bigcup \left(0, \frac{1}{5}\right]$.
Note that $x = 0$ is not included in either case - no division by 0!

(2) Consider the polynomials

$$p(x) = 2x^2 + 1$$
$$q(x) = x^3 - x + 1$$

(a) Write the product $(pq)(x)$ in expanded form (i.e. as a sum of terms, each of which is a constant times a power of x)

Solution:

$$\begin{aligned} (pq)(x) &= (2x^2 + 1)(x^3 - x + 1) \\ &= (2x^5 - 2x^3 + 2x^2) + (x^3 - x + 1) \\ &= 2x^5 - x^3 + 2x^2 - x + 1 \end{aligned}$$

(b) Write the composition $(p \circ q)(x)$ in expanded form.

Solution:

$$\begin{aligned} (p \circ q)(x) &= 2(x^3 - x + 1)^2 + 1 \\ &= 2(x^6 - x^4 + x^3 - x^4 + x^2 - x + x^3 - x + 1) + 1 \\ &= 2(x^6 - 2x^4 + 2x^3 + x^2 - 2x + 1) + 1 \\ &= 2x^6 - 4x^4 + 4x^3 + 2x^2 - 4x + 3 \end{aligned}$$

(3) Let L be the line containing the points $(1, 1)$ and $(5, 13)$.

(a) Find an equation for L.

Solution: First we'll compute the slope.

$$m = \frac{13-1}{5-1} = 124 = 3.$$

Choosing one of the points, say $(1, 1)$, we obtain L in point-slope form: $y-1 = 3(x-1)$.

(b) Find an equation for the line L' which is perpendicular to L and contains the origin $(0, 0)$.

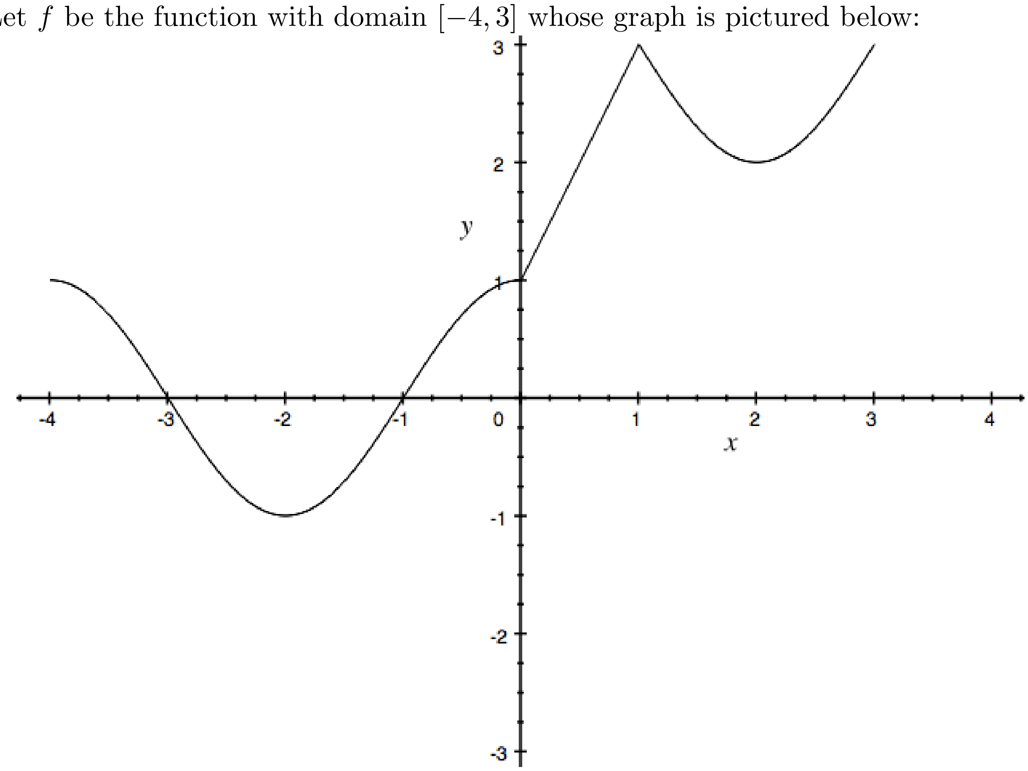
Solution: A line perpendicular to L has slope $-\frac{1}{3}$. We obtain L' in slope-intercept form: $y = -\frac{1}{3}x$.

(c) Find the point of intersection of L and L'.

Solution: We must find a point (x, y) which satisfies both equations simultaneously, i.e. $y = 3(x-1)+1$ and $y = -\frac{1}{3}x$. Setting the right sides equal,

$$\begin{aligned} 3(x-1)+1 &= -\frac{1}{3}x \\ 3x-3+1 &= -\frac{1}{3}x \\ -2 &= -\frac{10}{3}x \\ \frac{6}{10} &= x \\ x &= \frac{3}{5} \end{aligned}$$

(4) Let f be the function with domain $[-4, 3]$ whose graph is pictured below:



(a) What is the largest interval on which f is increasing?

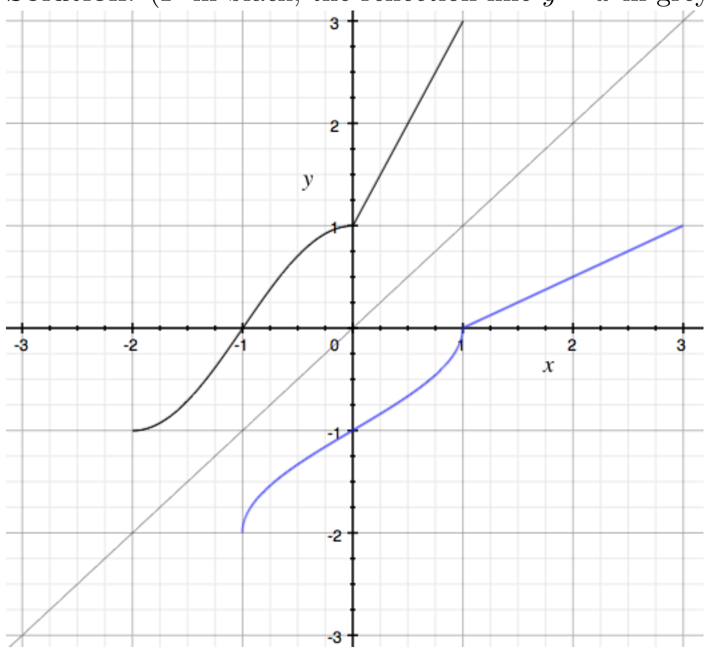
Solution: $[-2, 1]$.

(b) Let F be the function defined by restricting the domain of f to the interval from (a). Estimate the value of $F^{-1}(0)$.

Solution: There are two x-values (inputs of F) corresponding to the y-value (output of F) 0 on the graph of F, which I approximate to be -1 and -3. -3 is not in the restricted interval $[-2, 1]$, but -1 is. So $F^{-1}(0) \approx -1$.

(c) Sketch a graph of $F^{-1}(x)$.

Solution: (F in black, the reflection line $y = x$ in grey, F^{-1} in blue)



(5) Let $f(x) = 3x^2 - 2x + 1$.

(a) The graph of f is a parabola. Find the coordinates of the vertex of this parabola.

Solution: We'll complete the square.

$$\begin{aligned} f(x) &= 3\left(x^2 - \frac{2}{3}x + \frac{1}{3}\right) \\ &= 3\left(\left(x - \frac{1}{3}\right)^2 - \frac{1}{9} + \frac{1}{3}\right) \\ &= 3\left(\left(x - \frac{1}{3}\right)^2 + \frac{2}{9}\right) \\ &= 3\left(x - \frac{1}{3}\right)^2 + \frac{2}{3} \end{aligned}$$

The quadratic is now in vertex form, and we see that the vertex is $\left(\frac{1}{3}, \frac{2}{3}\right)$.

(b) Find all real solutions to the equation $f(x) = 0$.

Solution: It's not obvious how to factor this, so we'll use the quadratic equation.

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 3 \cdot 1}}{2 \cdot 3} = \frac{2 \pm \sqrt{-8}}{6}$$

But we can't take the square root of -8, so there are no real solutions.

Alternative Solution: Since the coefficient of x^2 is positive, the y-coordinate of the vertex, $\frac{2}{3}$, is the minimum value of f, so f never takes on the value 0. Put another way, the parabola lies entirely above the x-axis, so there are no x-intercepts.

(6) Let $f(x) = x^{-3} + 1$.

(a) Is f even, odd, or neither? Explain.

Solution: We'll test the value of $f(-x)$.

$$f(-x) = (-x)^{-3} + 1 = \frac{1}{(-x)^3} + 1 = -\frac{1}{x^3} + 1 = -(x^{-3}) + 1$$

This is not equal to $f(x)$, so f is not even.
But also this is not equal to $-f(x) = -(x^{-3}) - 1$, so f is not odd.
f is neither even nor odd.

(b) Sketch a graph of f.

Solution: This is the graph $y = x^{-3}$ (see the end of section 2.3 in the textbook) shifted up by 1 unit.