11/17/10 MIDTERM 3 REVIEW

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Topics:

(1) Optimization problems
- Write down all the information you are given, and (if appropriate) draw a picture which incorporates this information.
- Identify the relevant quantities, specifically the dependent quantity you are trying to maximize or minimize and the independent quantity which you can change. Find a formula for a relationship between these quantities.
- Find the derivative of the dependent quantity in terms of the independent quantity. Find critical points (zeros and discontinuities) of the derivative.
- Test the critical points and endpoints of the domain to find the max/min value.
- Example: Find an equation of the line through the point $(3, 5)$ that cuts off the least area from the first quadrant.

(2) L'Hospital's rule - quotients and other indeterminate forms
- You should be comfortable with the main form of L'Hospital's rule: $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$ for indeterminate forms $\frac{0}{0}$ and $\frac{\pm\infty}{\pm\infty}$.
- You should also know how to deal with other indeterminate forms: $\infty - \infty$, $0 \cdot \infty$, 0^0, ∞^0, and 1^∞. These can be solved by putting terms over a common denominator (indeterminate sum), taking the reciprocal of one term (indeterminate product), or taking logarithms (indeterminate powers).
- Sometimes it is necessary to use L'Hospital's rule multiple times or to manipulate
- Beware of forms which look indeterminate but are not. For example: $\infty + \infty$, $\frac{0}{\pm\infty}$, $\frac{\pm\infty}{0}$.
- Example: $\lim_{x\to a^+} \frac{\cos(x)\ln(x-a)}{\ln(e^x - e^a)}$. Applying L'Hospital's rule right away turns this into a mess. Solve this by separating into a limit of $\cos(x)$ times a limit of the quotient of logarithms.
- Example: $\lim_{x\to\infty} \left(1 + \frac{3}{x} + \frac{5}{x^2}\right)^x$.

(3) Sketching functions
- Recall the techniques we have learned through the semester. These techniques are roughly divided into three categories:
- Precalculus tools. Find the domain on which the function is defined. Find the y-intercept by plugging in 0 for x, and find the x-intercept by solving for x if possible. Look for any symmetries (is the function odd? even? periodic?).
- Limits. Find horizontal asymptotes by looking at the limits as x goes to ∞ and $-\infty$. Find vertical asymptotes by checking to see if $\lim_{x\to a^\pm} f(x) = \pm\infty$ for any a. Slant asymptotes: if the function f is given as a quotient and the numerator has 1 higher "degree in x", use polynomial long division to write f as a line plus a term that goes to 0 as x goes to $\pm\infty$.
- Derivative tests. Use the first derivative to find local maxima and minima and regions on which the function is increasing and decreasing. Use the second derivative to find points of inflection and regions on which the function is concave up and concave down.
- Example: Sketch the graph of $y = \frac{(x+1)^3}{(x-1)^2}$.

(4) Newton's method
- Know the formula: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.
- Know what is does: the new approximation (x_{n+1}) is the root of the linear approximation (the tangent line) to the function at the latest approximation (x_n).

(5) Antiderivatives
- Know the power rule: the antiderivative of x^n is $\frac{1}{n+1}x^{n+1}$.
- At this point, the best strategy we have is just to manipulate the function until it looks like the derivative of a function we know.
- The u-substitution method will not be covered on this midterm.
- Antidifferentiation can be used to solve word problems involving rate of change.
- Example: Let's say you start working on your homework at 8pm, and your productivity is measured by $\frac{dP}{dt} = -6t^2 + 12t$. How many problems do you get done between 9pm and 10pm?

(6) Definite integrals and Riemann sums
- Make sure you understand the connection between integral and area.
- You should be able to compute an approximation to the area under a curve using a Riemann sum with a small number of rectangles (say 3 or 4).

(7) Properties of integrals
- Review the list of integral properties that we derived. Make sure you understand their graphical interpretations.
- Make sure you are clear on the two ways that integrals count "negative area": area under the x-axis, and area counted right to left (when the upper bound of integration is less than the lower bound).
- One important property is the integral inequality theorem, which says that if $f(x) \leq g(x)$ on $[a, b]$, then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$.
- Example: Show that $\int_0^1 \sqrt{1+x^2}dx \leq \int_0^1 \sqrt{1+x}dx$.

(8) Fundamental theorem of calculus
- Part 1: You should be able to take the derivative of a function defined by an integral, using the chain rule and rewriting the integral if necessary.
- Example: $\frac{d}{dx}\int_{\sin(x)}^{\cos(x)}(1-x^2)dx$.
- Part 2: You should be able to calculate indefinite integrals using the antiderivative.
- Example: $\int_{-1}^1 |x^3|dx$.

(9) Indefinite integrals
- $\int f(x)dx$ means antiderivative of $f(x)$.
- Don't forget to add the C!
- Example: $\int 5x\sqrt{x}dx = 2x^{5/2} + C$.