10/20/10 MIDTERM 2 REVIEW

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Topics:

(1) Product rule
- Remember the rule, and know how to apply it.
- If things get too messy, look for another way out (trig identities, logarithmic differentiation, etc.)
- **Example**: Differentiate $x\ln(x) - x$.

(2) Quotient rule
- Same advice as for the product rule.
- **Example**: Find the tangent line to the graph of $\dfrac{x^2}{x^2-1}$ at $x = 2$.

(3) Limits involving $\lim_{x\to 0} \dfrac{\sin(x)}{x} = 1$
- If you can't figure out a limit involving a trig function, look for a place to apply this useful fact.
- Often it will be hidden - you'll have to rewrite trig function, use trig identities, or multiply the top and bottom by some value to put the limit in the right form.
- Remember that $\lim_{x\to a} \dfrac{\sin(f(x))}{f(x)} = 1$ as long as $f(x)$ goes to 0 as x goes to a. Usually $f(x)$ will be a constant multiple like $2x$. So $\lim_{x\to 0} \dfrac{\sin(2x)}{2x} = 1$.
- **Example**: Compute $\lim_{x\to 1} \dfrac{\tan(x^2-1)}{x-1}$.

(4) Derivatives of trig functions
- You should feel comfortable using the derivatives of the common trig functions in conjunction with the product, quotient, chain rules, etc.
- **Example**: Differentiate $f(x) = \sin^2(x)\tan(x)$.

(5) Chain rule
- Keep clear exactly what function you are differentiating at all times, and which functions are nested inside this function.
- Whenever you take the derivative of an expression, look hard for all the nested functions. It can be easy to miss things like constant multiples.
- Most chain rule mistakes come from sloppiness. Don't be afraid to break down complicated chains into multiple parts by substitution to keep things more manageable.
- **Example**: Differentiate $e^{\cos(\ln(2x))}$.

(6) Implicit differentiation
- Remember what variable you're differentiating in terms of - don't take $\frac{d}{dx}$ of the left side at $\frac{d}{dt}$ of the right.
- At every step, think: should I be applying the chain rule here?
- **Example**: Find the tangent line to the curve described by $e^{x/y} = x - y$ at the point $(0, -1)$.
- **Example**: Find $\frac{dy}{dx}$ if $y = \sin(x + y)$.

  (a) Differentiation of inverse functions
  - Use the rule derived in class and in the book for the derivative of an inverse function OR
  - Re-derive using implicit differentiation. Personally I find this easier to remember and think about. Write $f(f^{-1})(x) = x$ and use implicit differentiation.
  - **Example**: Find $(f^{-1})'(1)$ if $f(x) = e^x + \sin(x)$ and f is defined for $x \geq -1$.

(b) Logarithmic differentiation
- This can be useful when faced with complicated expressions involving products and exponents.
- Take the natural log of both sides and use implicit differentiation.
- **Example:** Differentiate $x^{\sin(x)}$.

(7) Related rates
- Write down all the information you are given, and draw a picture that incorporates this information. Getting the picture right is the most important step!
- Identify the quantity (usually a derivative) that you are trying to find.
- Identify which quantities are constant and which quantities are changing, and find their rates of change.
- Often a quantity will be changing, but we are asked to consider when it takes on a certain value (for example, the time t is changing, but we are asked to find how fast a ball is moving at time $t = 3$). Keep the distinction clear between these quantities and the constants.
- Look for relations between the important quantities - often these come from right triangles or geometry equations (area, circumference, surface area, volume).

(8) Linear approximations
- Pick a value for x at which the function is easy to evaluate.
- Find the tangent line to the graph of the function at x, and plug in the new value of x.
- OR equivalently, if $\frac{dy}{dx} = f'(x)$, write $dy = f'(x)dx$ and estimate the change in y (dy) by plugging in x and the change in x (dx). Remember to add the original value of y.
- **Example**: Use differentials to estimate $\sqrt{4.1}$.
- **Example:** The radius of a circular disk is measured to be 20cm with a possible error of .2cm. Estimate the possible error in computing the area of the disk.

(9) Local and global maxima and minima
- Identify the quantity you want to minimize/maximize.
- Remember that maxima and minima only occur at critical numbers - where the derivative is 0 or undefined.
- Global maxima and minima can occur at local maxima and minima or at endpoints.
- **Example:** Find the local and global max and min values of $f(x) = x^3 - 12x$ on the interval $[-3, 5]$.

(10) Mean value theorem
- It is much more important to know what the mean value theorem **means** than to have the exact statement memorized (or written down):
- Given a function f differentiable on $[a, b]$, there is a point c in $[a, b]$ such that the slope of the tangent line at c, $f'(c)$, is the same as the slope of the secant line from $(a, f(a))$ to $(b, f(b))$.
- A useful special case is Rolle's Theorem, which says that if $f(a) = f(b)$, then there is some c in $[a, b]$ such that $f'(c) = 0$. This can be used to count how many times a function takes on a specific value (like 0).
- **Example:** Show that the equation $e^x = -x$ has exactly one real root.

(11) Sketching graphs using calculus

(a) First derivative test: increasing/decreasing and max/min

(b) Second derivative test: concave up/down and inflection points
- Identify the points of interest - the critical numbers of the function and its first derivative. These are the points at which the first and second derivatives are zero or undefined.
- For each interval between two points of interest, identify the sign of the first and second derivatives. A table helps to organize this information.
- Use this information to identify portions of the graph which are increasing/decreasing and concave up/down. Identify local maxima and minima and inflection points.
- **Example:** Sketch the curve $y = x^{1/x}$ for $x > 0$.
- **Example:** Sketch the curve $y = \sqrt[3]{x^3 - x}$.