

Math 113 independent research project

In this project, you will write a short expository essay on a topic of your choice. Virtually anything related to geometry is fair game – you may further investigate something we did in class, take one of the suggestions below, or come up with a project on your own. Your essay should involve some history and some explanation of mathematics, how much of each is up to you.

Deadlines and requirements:

- (1) Written proposal – you must e-mail the instructor what topic you plan to do and what you would like to do with it before tutorial this Thursday. We will discuss the project briefly 1-on-1 during tutorial time. I can help point you towards some more resources.
- (2) Final essay due: Friday February 24th (8th week)
- (3) Length: Different topics will require different treatment. As a rough guideline, you could aim for around 1500 words (about $\frac{2}{3}$ of the length of the reading on the classification of finite simple groups). More mathematics, rather than history, pictures/figures, or a more challenging topic might affect the word count.
- (4) Possible short oral presentation?

Some suggested topics:

- (1) **Proofs and Refutations**. Read the full dialogue (all 4 parts). What point is Lakatos making about the methodology of doing mathematics? Alternatively, what (in your opinion) makes a proof? How does evidence from the dialogue support/disagree with this?
- (2) **M.C. Escher** Write on the mathematics behind some of his work. (Tesselations/tilings, tilings of hyperbolic space...). There are many good internet resources on this. We will be discussing tilings in class this week.
- (3) **Polyhedra** Build a model (or some models) of regular, non-regular or star polyhedra with lots of symmetry. Discuss the symmetry group and the properties of the polyhedron. What is the dual polyhedron? Need more ideas? Try Buckyballs.
- (4) **Braid groups** (the kind where you pay attention to whether strands are crossed over or under). Write an expository article on braids showing that this is a group, and showing that something analogous to transpositions generates it. You might want to start by reading the first section of the wikipedia article on the braid group.

- (5) **The four color theorem.** If you want to color in countries in a map, it's generally a good idea not to make two neighboring countries the same color. The *four color theorem* says that any map (of any countries, made up or real) can be colored this way using only four different colors. (try it!) Though it sounds amazingly simple, this was only proved rather recently. The proof of the theorem is amazingly complicated and relies on some very heavy computer calculations. Write a short essay on the history of this problem and it's controversial computer-aided proof. This relates to "graph theory" which we will study at the end of the quarter.
- (6) **Four-dimensional space.** We're familiar with one, two and three-dimensional objects. What does a four-dimensional "polyhedron" (called a polytope) look like? Write a report on the history and/or mathematics of the fourth dimension. One suggestion of where to start is with the background information and/or videos of the first few chapters here: http://www.dimensions-math.org/Dim_E.htm
- (I have not watched these videos, but the background essays are very well done, and mathematician Etienne Ghys is a wonderful expositor)
- (7) **Non-euclidean geometry.** You probably think it's always true that two parallel lines never meet and always stay the same distance apart (isn't that the definition of parallel?) In the world of non-euclidean geometry, it makes sense for "parallel" lines to approach, diverge, or sometimes even cross. Learn and write about some of the history of this (once controversial, now commonplace) subject.
- (8) **Fractals** ...are self-similar shapes. They're beautiful (google image search "fractal"), found in nature (why? how?) and of course, mathematical. Some interesting things to explore are: shapes with finite area but infinite perimeter, self-similarity, "dimension" in between 1 and 2... There are many good introductory books available.
- (9) **Wallpaper groups.** Classifying the ways to tile the plane. Write a short history or expository essay. We'll talk about these a bit in class, so you can start with that section of your course notes book.
- (10) **Topic of your choice!** For this, you should provide a few more details in your topic proposal. Feel free to e-mail the instructor ahead of time and ask questions, or ask for more suggestions.