further study of topology, appears at the end of the book. Among the books and articles that had the greatest influence on this book, I would like to name the book by Rolphsen [3] and the article by Viro [5].

As is usually done in mathematical books and papers, the symbol □ marks the end of the proof of a proposition or a theorem.

It should be mentioned that the present text is based on a series of lectures given by the author in the academic year 1990–1991 to students of High School no. 57 in Moscow.

I am grateful to N. M. Fleischer for useful discussions of the manuscript.

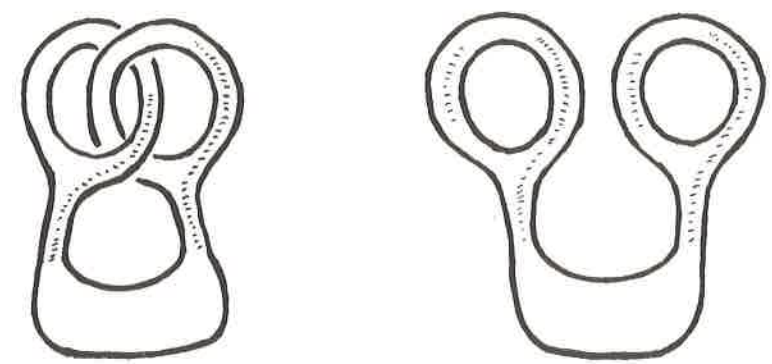
# 1

# Deformations

Our first look at topology will involve some problems about the deformation of elastic bodies and surfaces. We shall assume that the objects considered are made from a very elastic material: their shape may be changed at will, you can bend, distort, stretch, and compress them as much as you like, but of course you may not tear them or glue parts of them together.

The deformations that you will be asked to find will seem impossible at first glance. But actually they are not difficult to visualize, as you can verify by reading their description in the solution section. However, we emphatically suggest that you try to find the solution on your own before looking at our answers.

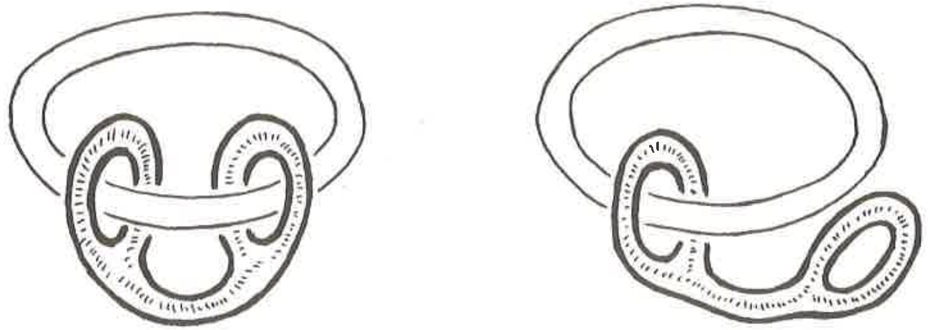
*Problem* 1.1. Show that the elastic body represented in Figure 1.1 (a) can be deformed so as to become the one shown in Figure 1.1 (b). In other words, were the human body elastic enough, after making linked rings with your index fingers and thumbs, you could move your hands apart without separating the joined fingertips.



(a) (b)

FIGURE 1.1

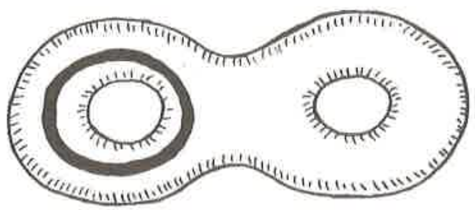
*Problem* 1.2. A pretzel has two holes that "hold" a doughnut (see Figure 1.2 (a)). Show that the pretzel can be deformed in such a way that one of its "handles" will unlink itself from the doughnut (Figure 1.2 (b)).



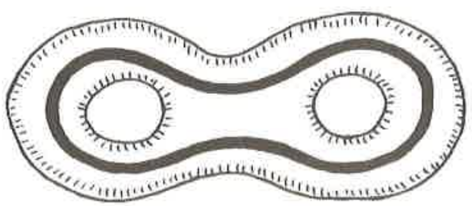
(a) (b)

FIGURE 1.2

*Problem* 1.3. A circle is drawn on à pretzel with two holes (Figure 1.3 (a)). Show that it is possible to deform the pretzel so that the circle will be in the position represented in Figure 1.3 (b).



(a)

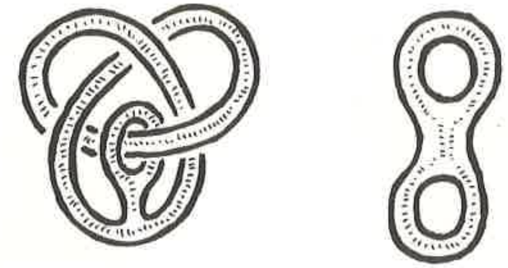


(b)

FIGURE 1.3

*Problem* 1.4. Show that a punctured tube from a bicycle tire can be turned inside out. (More precisely, this would be possible if the rubber from which the tube is made were elastic enough. In real life it is impossible to turn a punctured tube inside out.)

*Problem* 1.5. Show that the fancy pretzel represented in Figure 1.4 (a) can be deformed into the ordinary pretzel with two holes (Figure 1.4 (b)).



(a) (b)

FIGURE 1.4

**Solutions.**

We present most of the solutions to the problems in this book by means of pictures, which, as a rule, are self-explanatory. We sometimes indicate by arrows on the pictures the direction of motion or of deformation.

1.1. See Figure 1.5. We shall return to this deformation in §4.

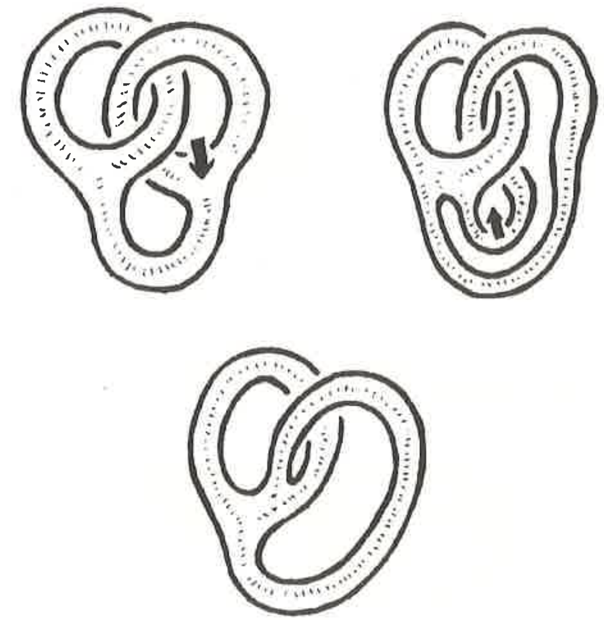


FIGURE 1.5

1.2. See Figure 1.6.

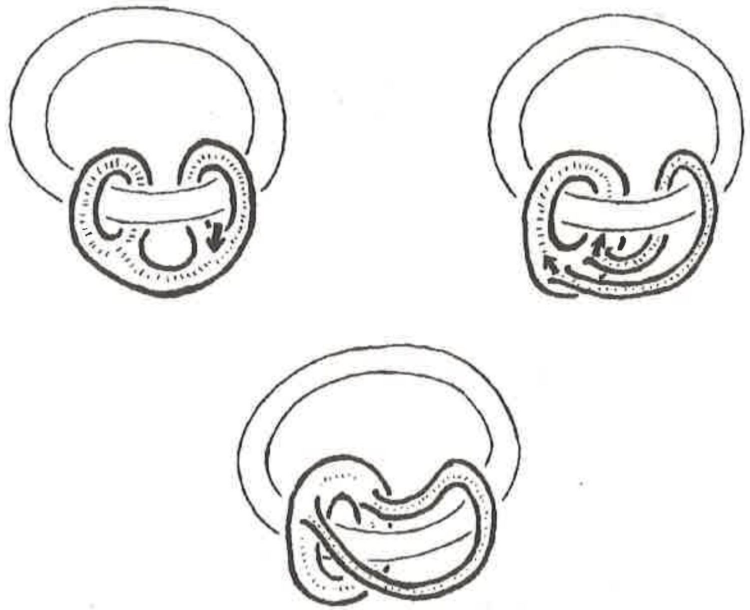


FIGURE 1.6

1.3. See Figure 1.7.

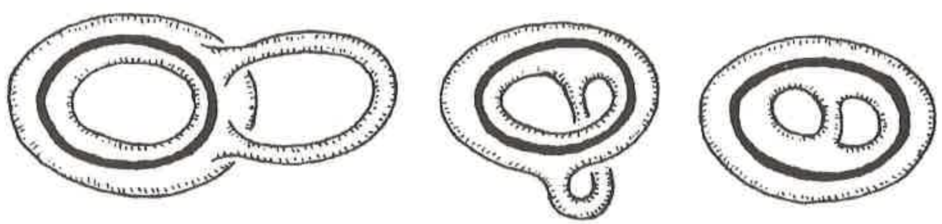


FIGURE 1.7

1.4. First we perform the deformations shown in Figure 1.8.

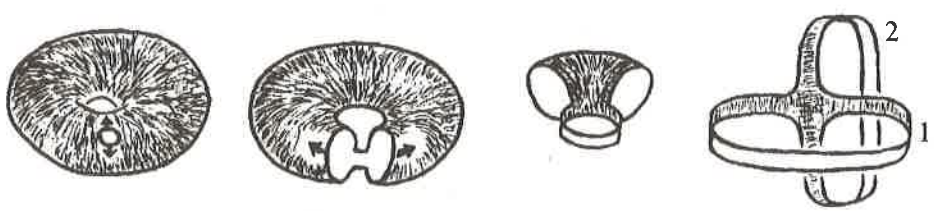



FIGURE 1.8



Then we can change the position of the obtained figure so that its "inside" (shown in white) becomes its "outside" (the shaded side of the surface) and vice-versa, simply by moving it as a rigid body in space until the hoop 1 occupies the position of the hoop 2. Once this is done, the previous deformations performed in reverse order result in the tube being turned inside out as required.

Note that this procedure interchanges the "parallel" and the "meridian" of the tube (see Figure 1.9).

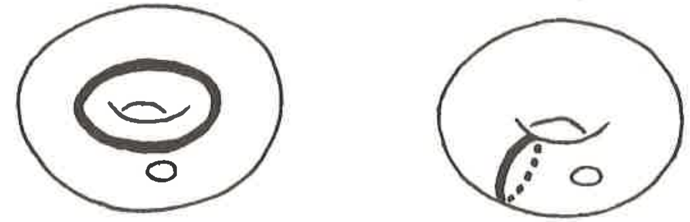


FIGURE 1.9

1.5. First we perform the deformation shown in Figure 1.10. The solid thus obtained (provided it is elastic) can clearly be deformed into the one shown in Figure 1.1 (a). It now remains to apply the solution of Problem 1.1.

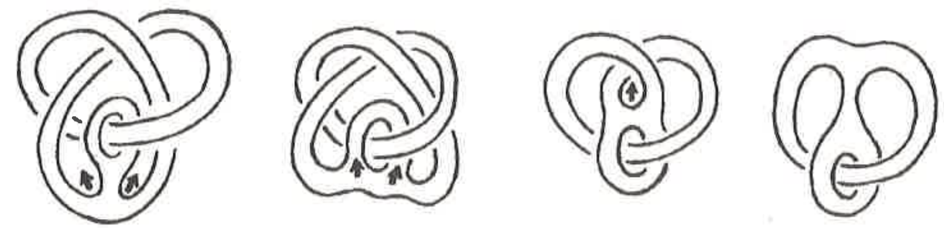


FIGURE 1.10