Math 113 Review for test 2

Material covered
You are responsible for all of the material *on rings* covered in class and on homework up until (but not including) the lecture on Thursday, November 13.

The relevant sections of the textbook are as follows: 7.1, 7.2 not including the part on "group rings", 7.3 (we didn't talk about isomorphism theorems 2, 3 and 4), 7.4. Instead of 7.5, there is a reading about fields of fractions on the course website. 7.6, 8.1, and 8.2 but not including the part on "Dedekind-Hasse norms."

I expect you to be able to define or explain each of these terms, and *give an example* where appropriate.

> Ring, subring, division ring, zero divisor, integral domain, unit, field, ring homomorphism (and isomorphism), kernel of a ring homomorphism, ideal (left, right, or two-sided), quotient ring, "ideal generated by...", principal ideal, prime ideal, maximal ideal, field of fractions, greatest common divisor (in a ring, not just for integers!), Euclidean algorithm, Euclidean domain, norm (for an integral domain), principal ideal domain.

Other words you encountered that I will certainly remind you of the definition (but you should be familiar with them):

> IJ, $I + J$ (when I and J are ideals of a ring), comaximal, the first isomorphism theorem for rings, quadratic integer ring, quadratic field.

Practice problems

1. **Make sure you can do all the problems on the homework**

2. Describe all elements of the ideal generated by $x^2 + 1$ in $\mathbb{Z}[x]$. Is it prime? Is it maximal? What about the ideal generated by $x^2 - 4$? Describe the quotient rings $\mathbb{Z}[x]/I$ when I is either of the ideals in this question.
3. Give an example to show that greatest common divisors are not necessarily unique
4. Find a G.C.D. of $x^2 - 2x + 1$ and $5x^3 - 2x - 3$ in $\mathbb{Z}[x]$. How do you know it is a GCD?
5. Give an example of two *comaximal* ideals in your favorite polynomial ring
6. Give an example of an ideal that is prime but not maximal.
7. Give an example of an integral domain that is not a Euclidean domain.

Problems from DF ...all the questions that were on previous homework plus
Section 7.1 # 7, 12, 13a, 14, 18
Section 7.2 # 1b, 2
Section 7.3 # 5, 7, 10, 12a,b
Section 7.4 # 4, 12, review the parts of 14-16 that were on HW

Section 7.5 – No problems from this section. Instead, try to prove the following:
If K is a field, then the field of fractions of K is isomorphic to K.
A proof is given in the extra reading on the webpage.

Section 7.6 # 5
Section 8.1 # 1c, 3, 5
Section 8.2 # 7a